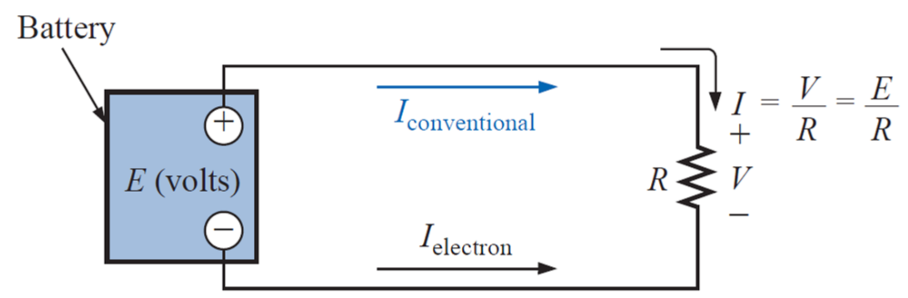
Series Circuits

Two types of current are readily available to the consumer today. One is *direct current* (dc), in which ideally the flow of charge (current) does not change in magnitude (or direction) with time. The other is *sinusoidal alternating current* (ac), in which the flow of charge is continually changing in magnitude (and direction) with time.

The battery of Fig. below, by virtue of the potential difference between its terminals, has the ability to cause (or "pressure") charge to flow through the simple circuit. The positive terminal attracts the electrons through the wire at the same rate at which electrons are supplied by the negative terminal. As long as the battery is connected in the circuit and maintains its terminal characteristics, the current (dc) through the circuit will not change in magnitude or direction.



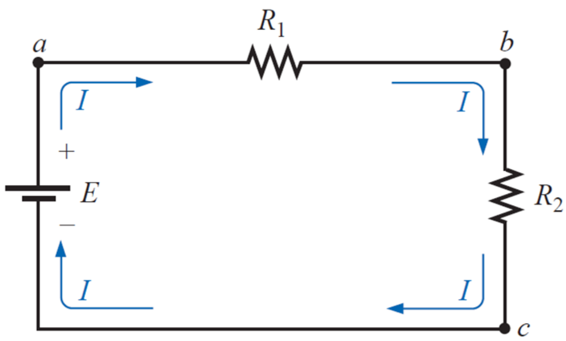
Introducing the basic components of an electric circuit.

The SERIES CIRCUIT consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Fig. 5.4(a) has three elements joined at three terminal points (a, b, and c) to provide a closed path for the current I.

Two elements are in series if

1. They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).

2. The common point between the two elements is not connected to another current-carrying element.



In series circuits

The total resistance of a series circuit is the sum of the resistance levels.

The current is the same through each element.

$$R_T = R_1 + R_2 + \cdots \ldots \ldots + R_N$$

$$I_S = I_1 = I_2 = \cdots \ldots \ldots = I_N$$

using Ohm's law; that is,

$$V_1 = IR_1 , V_2 = IR_2 , V_3 = IR_3 , \ldots . . V_N = IR_N$$

The power delivered to each resistor can be calculated as

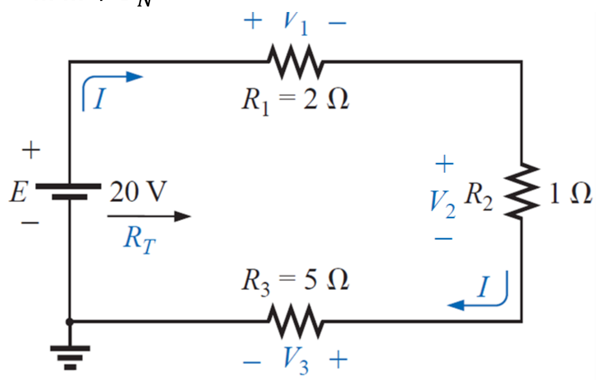
$$P_1 = IV_1 = \frac{V_1}{R_1} V_1 = \frac{{V_1}^2}{R_1} = I^2 R_1$$

The total power delivered to a resistive circuit is equal to the total power dissipated by the resistive elements.

$$P_{del} = P_1 + P_2 + \cdots \dots \dots + P_N$$

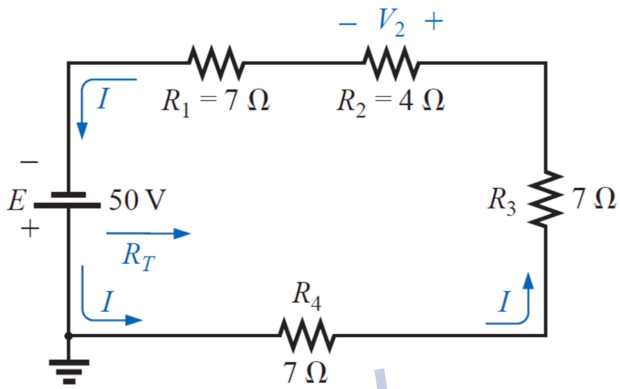
EXAMPLE 1

a. Find the total resistance for the series circuit.
b. Calculate the source current *Is*.
c. Determine the voltages *V*1, *V*2, and *V*3.
d. Calculate the power dissipated by *R*1, *R*2, and *R*3.
e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).



EXAMPLE 2

Determine R_T, I, and V_2 for the following circuit



Electromotive Force (EMF)

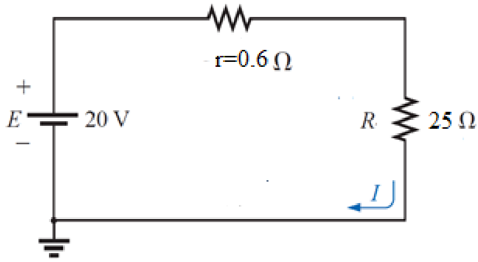
$E = V + Ir$

$V = E - Ir$

$E = IR + Ir = I(r + R)$

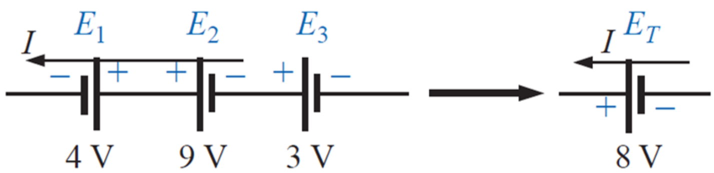
EXAMPLE 3

Calculate the EMF of the cell in figure shown below



VOLTAGE SOURCES IN SERIES

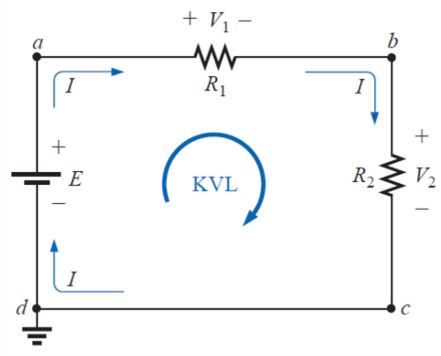
Voltage sources can be connected in series, , to increase or decrease the total voltage applied to a system.



KIRCHHOFF'S VOLTAGE LAW

Kirchhoff's voltage law ***(KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.***
A **closed loop** is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

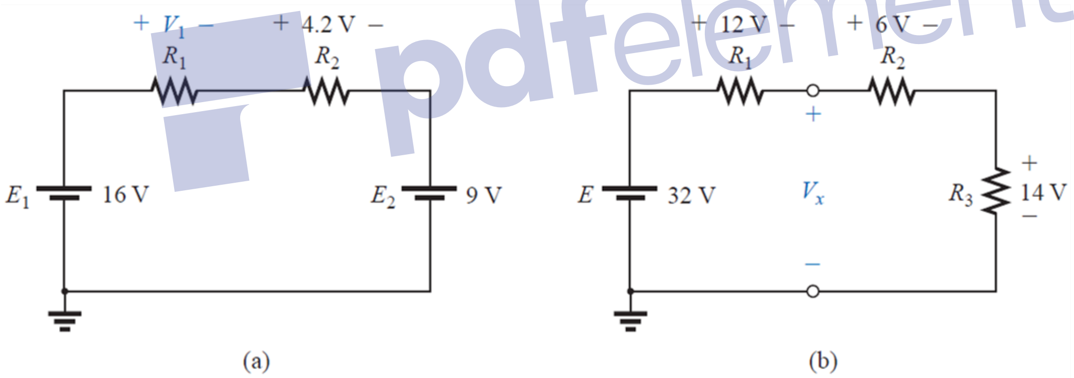
$$\sum_{\circlearrowright} V = 0$$



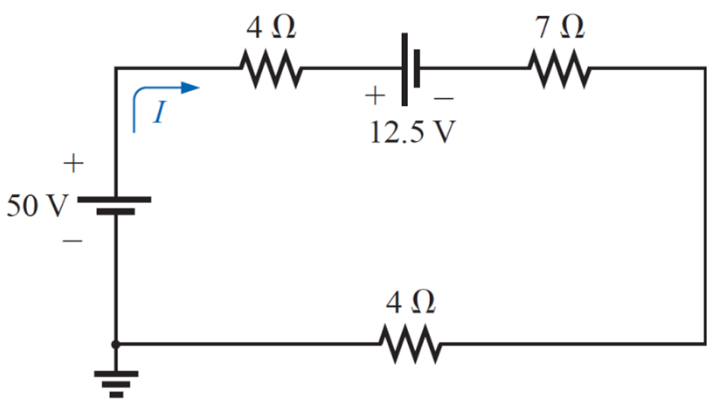
Kirchhoff's voltage law can also be stated in the following form:

$$\sum_{\circlearrowright} V_{\text{rises}} = \sum_{\circlearrowright} V_{\text{drops}}$$

EXAMPLE 4
Determine the unknown voltages for the networks of following Figures



EXAMPLE 5
Determine *I* and the voltage across the 7-_ resistor for the network of the Figure shown below.



VOLTAGE DIVIDER RULE

In a series circuit,
the voltage across the resistive elements will divide as the magnitude of the resistance levels.

$R_T = R_1 + R_2$

$I = \frac{E}{R_T}$

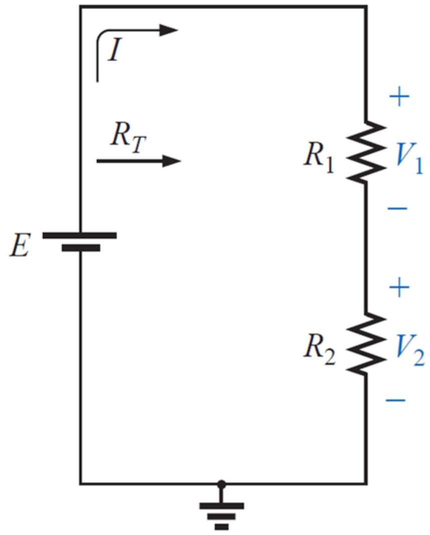
Applying Ohm's law:

$V_1 = IR_1 = \frac{E}{R_T} R_1 = E\frac{R_1}{R_T}$

$V_2 = IR_2 = \frac{E}{R_T} R_2 = E\frac{R_2}{R_T}$

In general

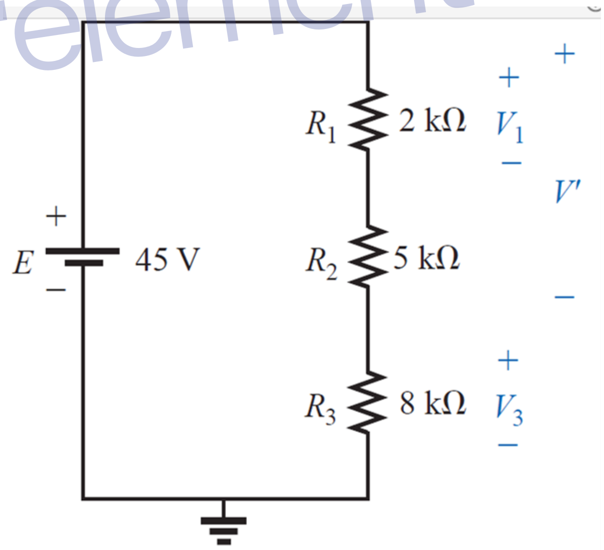
$V_x = E\frac{R_x}{R_T}$



In words, the **voltage divider rule** states that
The voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.

EXAMPLE 6

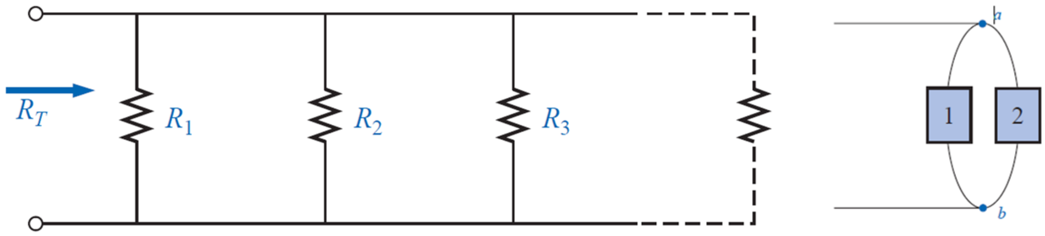
Using the voltage divider rule, determine the voltages V_1 and V_3 for the series circuit shown below



Parallel Circuits

PARALLEL ELEMENTS

Two elements, branches, or networks are in parallel if they have two points in common.



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \ldots + \frac{1}{R_N}$$

$$G_T = G_1 + G_2 + \cdots \ldots G_N$$

In parallel circuits

The voltage across parallel elements is the same.

Using this fact will result in

$E = V_1 = V_2$

but

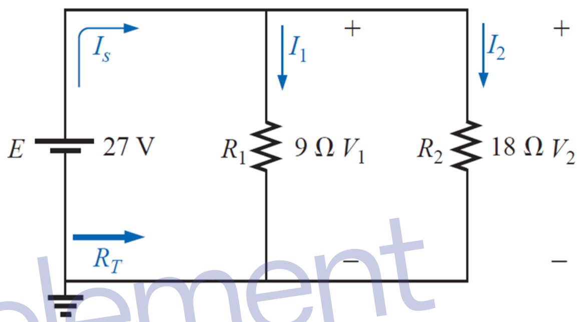
$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$

$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2}$

$\frac{E}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$

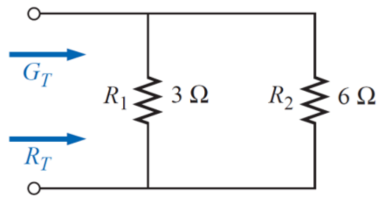
$I_s = I_1 + I_2$

Therefore, ***The total current equal to algebraic sum of branches current***



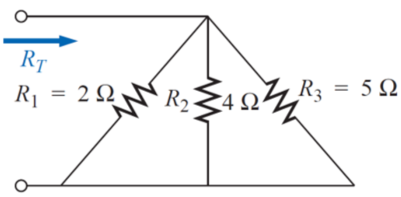
EXAMPLE 1

Determine the total conductance and resistance for the parallel network of Fig. shown below



EXAMPLE 2

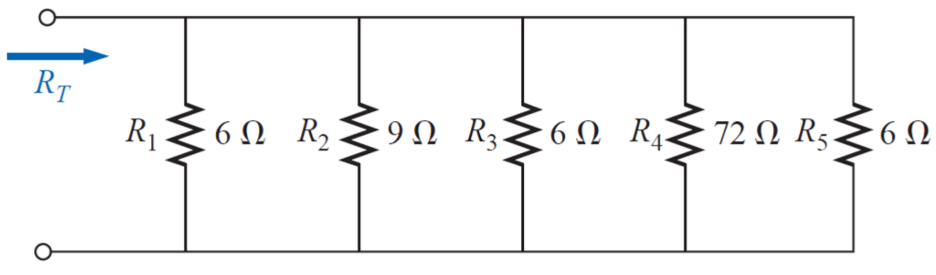
Determine the total conductance and resistance for the parallel network of Fig. shown below



The total resistance of parallel resistors is always less than the value of the smallest resistor.

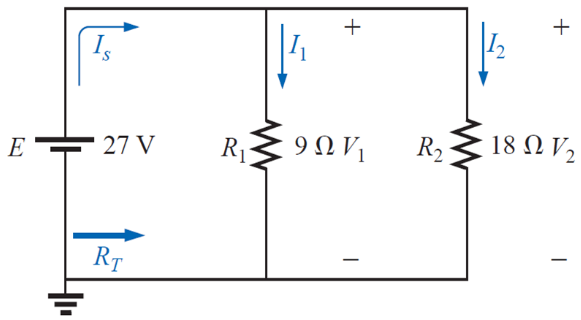
EXAMPLE 3

Determine the total conductance and resistance for the parallel network of Fig. shown below



EXAMPLE 4

For the following parallel network:



a. Calculate R_T.
b. Determine I_s.
c. Calculate I_1 and I_2, and demonstrate that $Is = I_1 + I_2$.
d. Determine the power to each resistive load.
e. Determine the power delivered by the source, and compare it to the total power dissipated by the resistive elements.

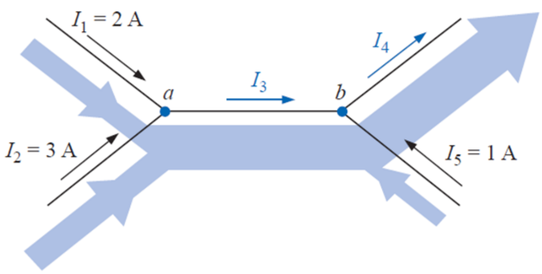
KIRCHHOFF'S CURRENT LAW

Kirchhoff's voltage law provides an important relationship among voltage levels around any closed loop of a network. We now consider
Kirchhoff's current law (KCL), which provides an equally important relationship among current levels at any junction.
Kirchhoff's current law (KCL) ***states that the algebraic sum of the currents entering and leaving an area, system, or junction is zero.***
In other words,
the sum of the currents entering an area, system, or junction must equal the sum of the currents leaving the area, system, or junction.
In equation form:

$$\Sigma\ I_{\text{entering}} = \Sigma\ I_{\text{leaving}}$$

EXAMPLE 5

Determine the currents I_3 and I_4

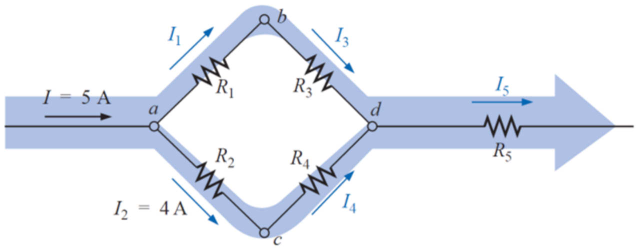


EXAMPLE 6

Find the magnitude and direction of the currents I_3, I_4, I_6, and I_7 for the network of following Figure. Even though the elements are not in series or parallel, Kirchhoff's current law can be applied to determine all the unknown currents.

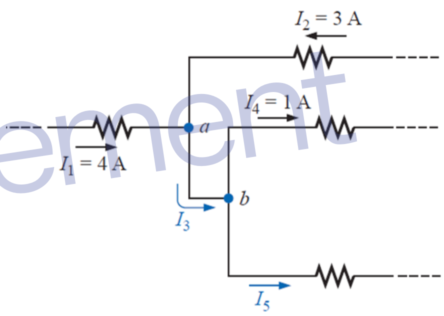
EXAMPLE 7

Determine I_1, I_3, I_4, and I_5 for the network shown below

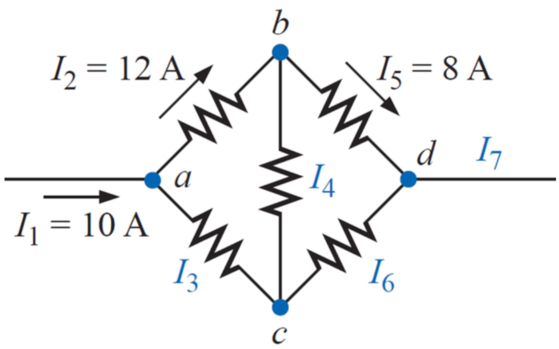


EXAMPLE 8

Determine the currents I_3 and I_5 of Fig. shown below through applications of Kirchhoff's current law.



EXAMPLE 9 Find the magnitude and direction of the currents I_3, I_4, I_6, and I_7 for the network shown below. Even though the elements are not in series or parallel, Kirchhoff's current law can be applied to determine all the unknown currents.



CURRENT DIVIDER RULE

The **current divider rule (CDR)** will determine how the current entering a set of parallel branches will split between the elements.

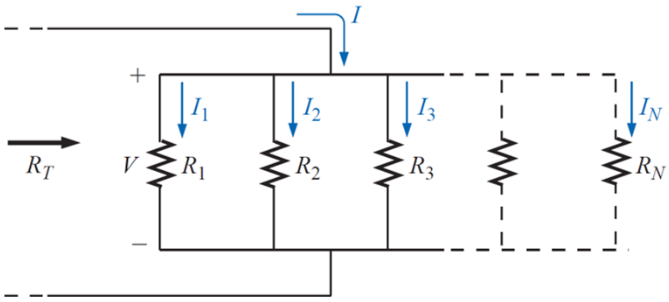
For two parallel elements of equal value, the current will divide equally.

For parallel elements with different values,

The smaller resistance has greatest value of current.

The current will split with a ratio equal to the inverse of their resistor values.

$$I_T = I_1 + I_2 + \cdots + I_N$$
$$E = V_1 = V_2 = V_N$$
$$I_T = \frac{E}{R_T} = \frac{I_1 R_1}{R_T} = \frac{I_2 R_2}{R_T} = \frac{I_N R_N}{R_T}$$
$$I_T = \frac{I_x R_x}{R_T}$$
$$I_x = I_T \frac{\frac{1}{R_x}}{\frac{1}{R_T}} = I_T \frac{R_T}{R_x}$$

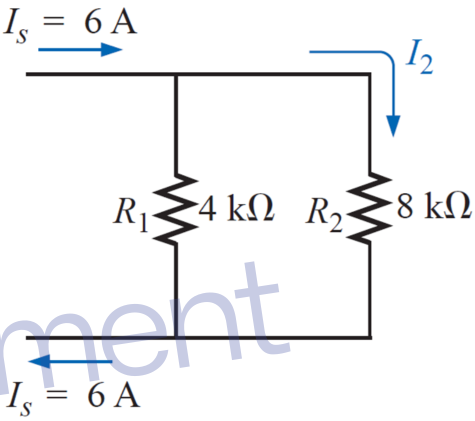


For the particular case of *two parallel resistors,*

$$I_1 = I_T \frac{\frac{1}{R_1}}{\frac{1}{R_T}} = I_T \frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}} = I_T \frac{\frac{1}{R_1}}{\frac{R_1+R_2}{R_1 R_2}} = I_T \frac{R_2}{R_1+R_2}$$
$$I_2 = I_T \frac{\frac{1}{R_2}}{\frac{1}{R_T}} = I_T \frac{\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}} = I_T \frac{\frac{1}{R_2}}{\frac{R_1+R_2}{R_1 R_2}} = I_T \frac{R_1}{R_1+R_2}$$

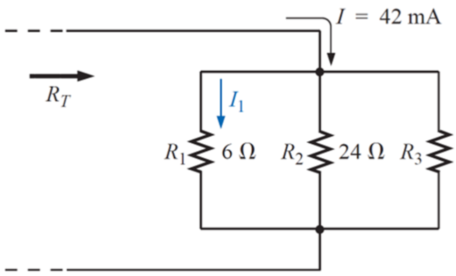
EXAMPLE 10

Determine the current I_2 for the network shown below using the current divider rule.



EXAMPLE 11

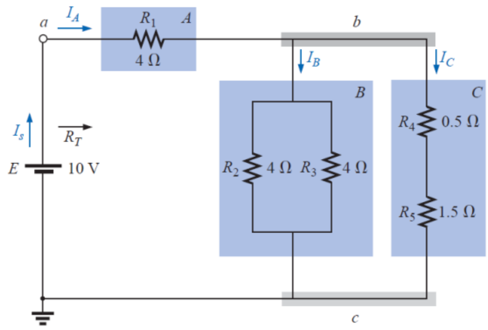
Determine the current I_1 for the network shown below using the current divider rule.



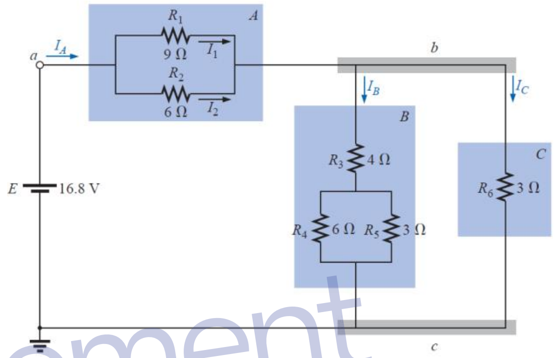
Series-Parallel Networks

series-parallel networks ***are networks that contain both series and parallel circuit configurations.***

Example 1
Find the indicated currents of the figure shown below

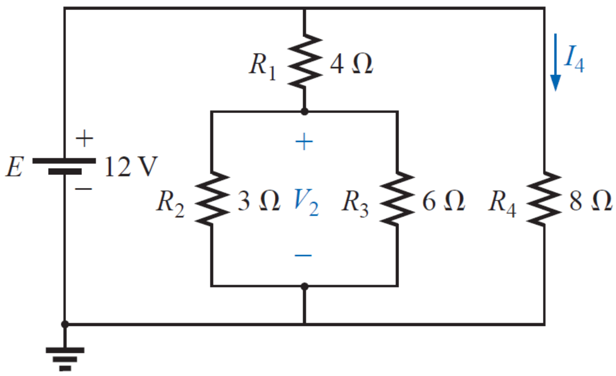


Example 2
Find the indicated currents of the figure shown below

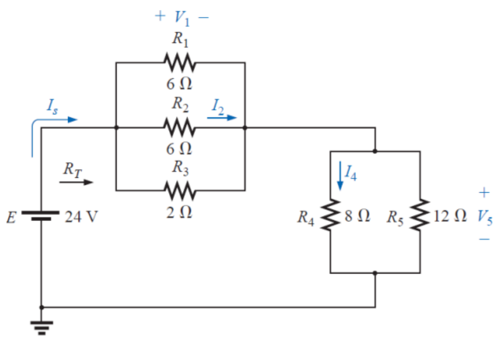


EXAMPLE 3

Find the current I_4 and the voltage V_2 for the network shown below

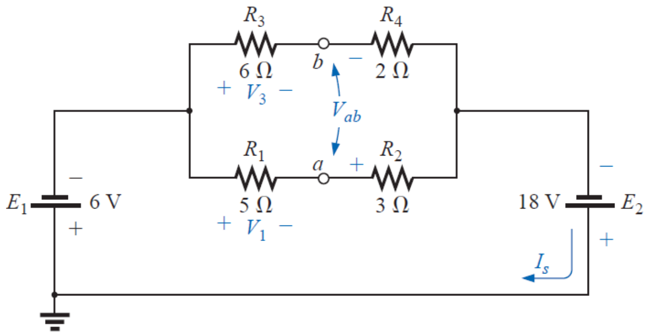


Example 4
Find the indicated currents and voltages for the network shown below



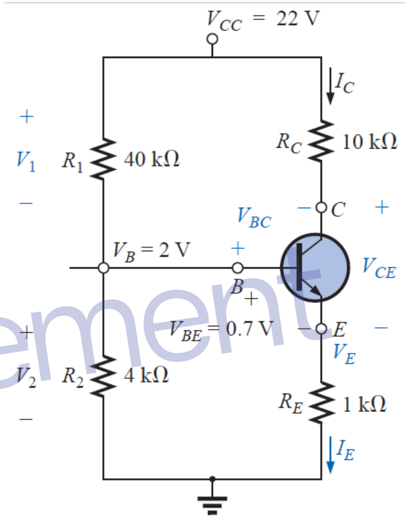
EXAMPLE 5

a. Find the voltages V_1, V_3, and V_{ab} for the network shown below.
b. Calculate the source current I_s.

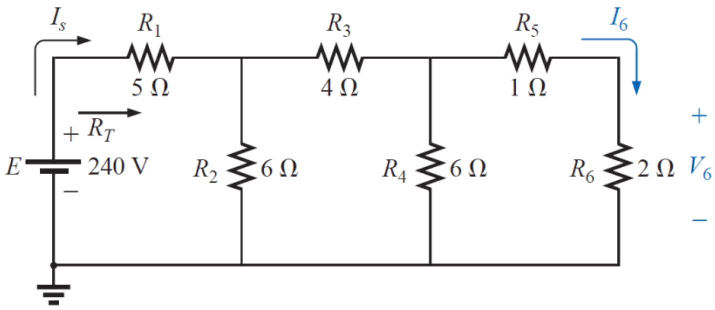


EXAMPLE 6

For the transistor configuration shown below, in which V_B and V_{BE} have been provided:
a. Determine the voltage V_E and the current I_E.
b. Calculate V_1.
c. Determine V_{BC} using the fact that the approximation $I_C = I_E$ is often applied to transistor networks.
d. Calculate V_{CE} using the information obtained in parts (a) through (c).



Example 7
Find the indicated currents of the figure shown below

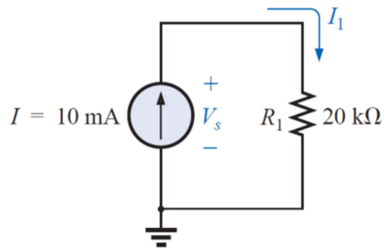


Methods of Analysis and Selected Topics (dc)

CURRENT SOURCES

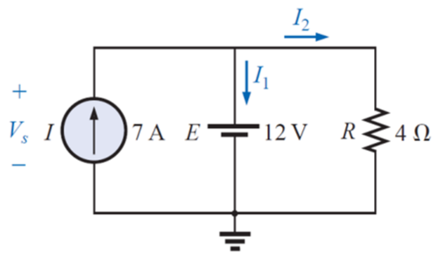
EXAMPLE 1

Find the source voltage V_s and the current I_1 for the circuit shown below

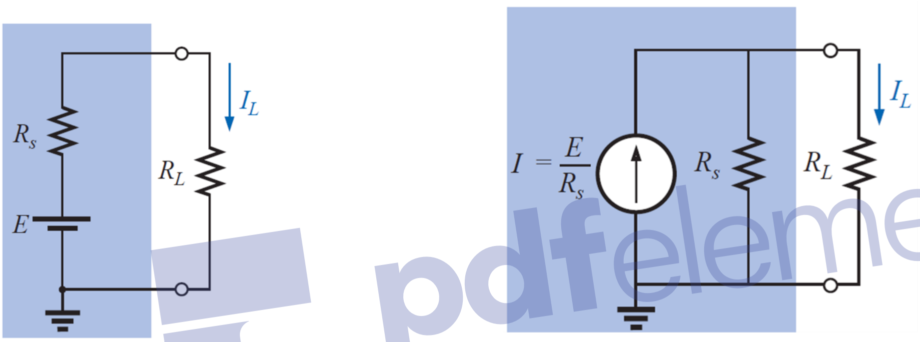


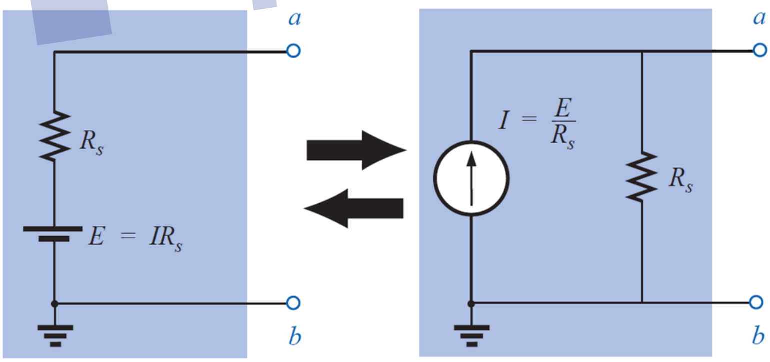
EXAMPLE 2

Find the source voltage V_s and the current I_1 for the circuit shown below



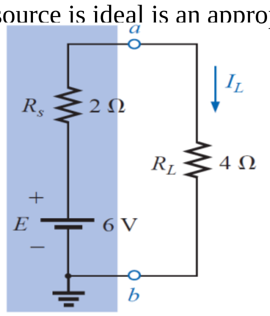
SOURCE CONVERSIONS

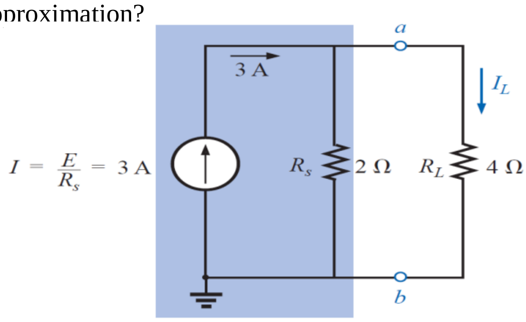




EXAMPLE 3

a. Convert the voltage source of Fig. 8.9(a) to a current source, and calculate the current through the 4Ω load for each source.
b. Replace the 4Ω load with a 1-kΩ load, and calculate the current I_L for the voltage source.
c. Repeat the calculation of part (b) assuming that the voltage source is ideal (R_s =0 Ω) because R_L is so much larger than R_s. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?



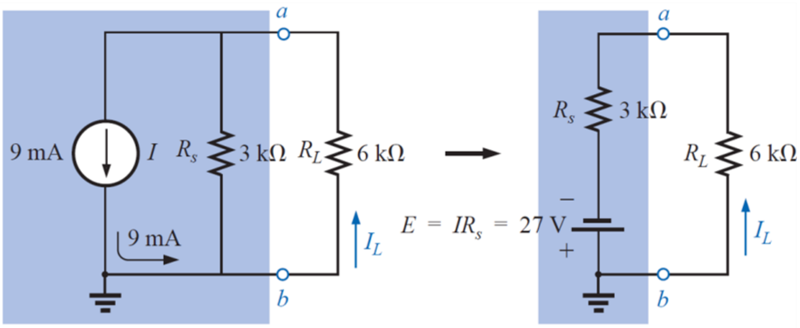


EXAMPLE 4

a. Convert the current source of Fig. shown below to a voltage source, and find the load current for each source.

b. Replace the 6-kΩ load with a 10Ω load, and calculate the current I_L for the current source.

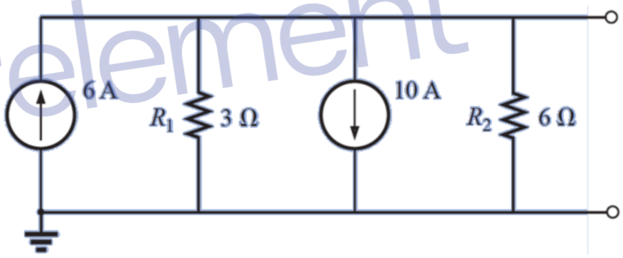
c. Repeat the calculation of part (b) assuming that the current source is ideal ($Rs = \infty\ \Omega$) because R_L is so much smaller than R_s. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?



(a) (b)

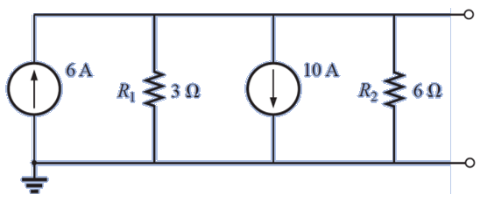
CURRENT SOURCES IN PARALLEL

If two or more current sources are in parallel, they may all be replaced by one current source having the magnitude and direction of the resultant, which can be found by summing the currents in one direction and subtracting the sum of the currents in the opposite direction.



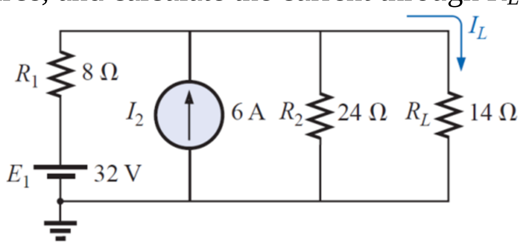
EXAMPLE 5

Reduce the parallel current sources of Fig. shown below to a single current source.



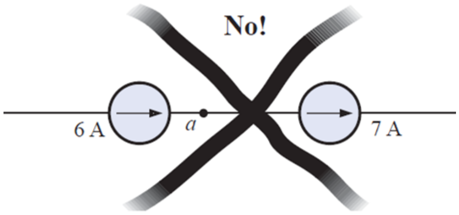
EXAMPLE 6

Reduce the network of Fig. shown below to a single current source, and calculate the current through R_L.



CURRENT SOURCES IN SERIES

current sources of different current ratings are not connected in series,

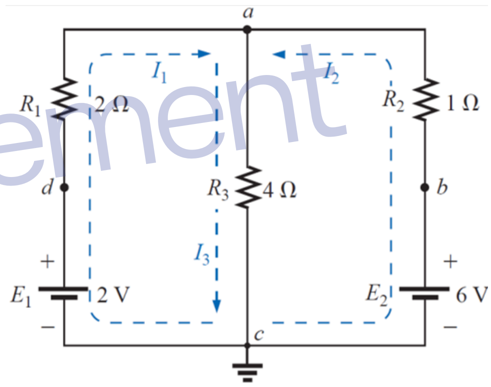


BRANCH-CURRENT ANALYSIS

1. ***Assign a distinct current of arbitrary direction to each branch of the network (N).***
2. ***Label each of the N branch currents.***
3. ***Indicate the polarities for each resistor as determined by the assumed current direction.***
4. ***Count the number of current sources***
5. ***Number of variables equal to N- number of current sources***
6. ***Apply Kirchhoff's voltage law around each closed, independent loop of the network.***
7. ***Express any additional organize the equations.***
8. ***Solve the resulting simultaneous linear equations for assumed branch currents.***

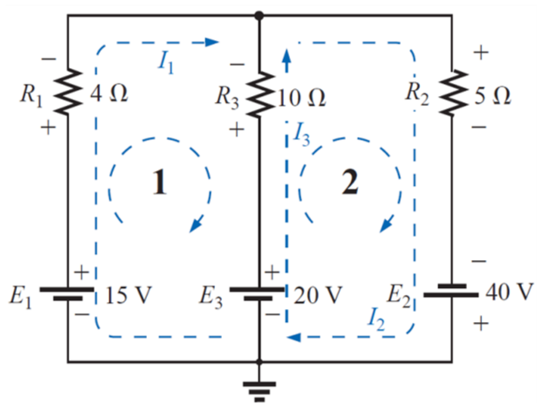
Example 7

Apply the branch-current method to the network of the following network



Example 8

Apply the branch-current method to the network of the following network

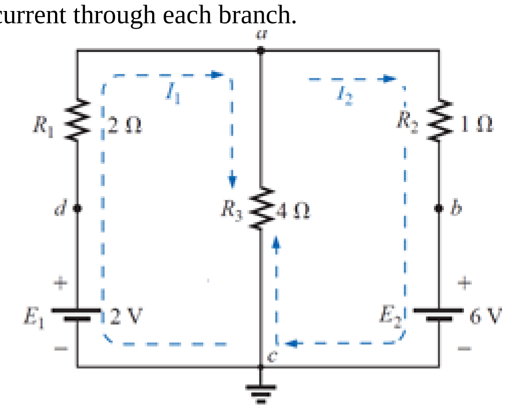


MESH ANALYSIS

1. ***Assign a distinct current in the clockwise or anticlockwise direction to each independent, closed loop of the network (N).***
2. ***Label each of the N mesh currents.***
3. ***Indicate the polarities for each resistor as determined by the assumed current direction.***
4. *Count the number of current sources*
5. *Number of variables equal to N- number of current sources*
6. ***Apply Kirchhoff's voltage law around each closed, independent loop of the network.***
7. ***Express any additional organize the equations.***
8. ***Solve the resulting simultaneous linear equations for assumed branch currents.***

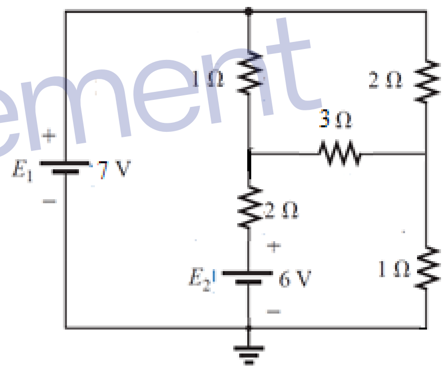
Example 9

Apply the Mesh method to the network of the following network to find the current through each branch.



Example 10

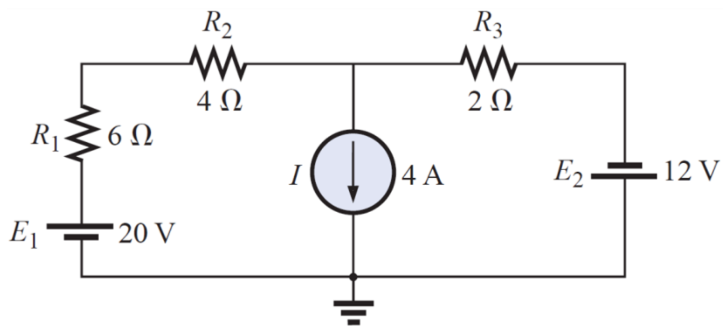
Find the branch currents of the network shown below



Supermesh Currents

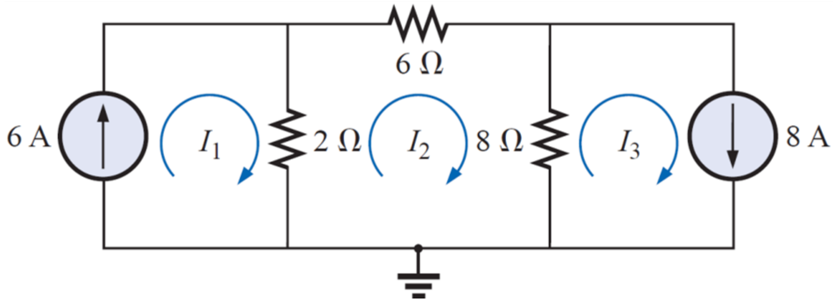
Example 11

Using mesh analysis, determine the currents of the network shown below



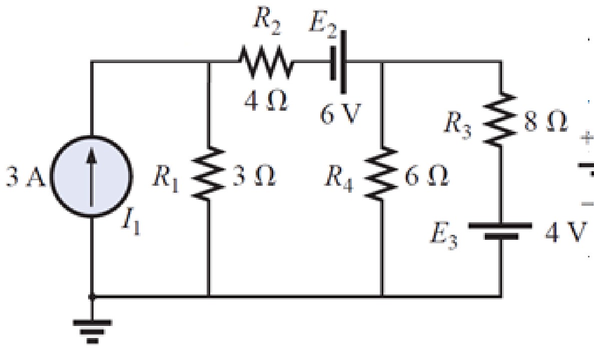
Example 12

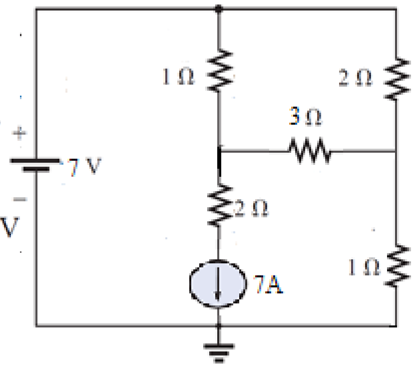
Using mesh analysis, determine the currents of the network shown below



H.W

Find the mesh currents of the network shown below



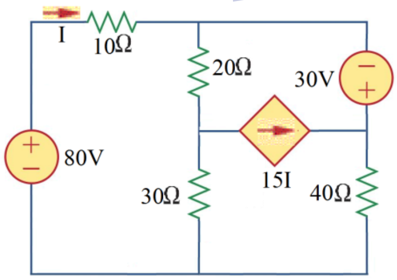


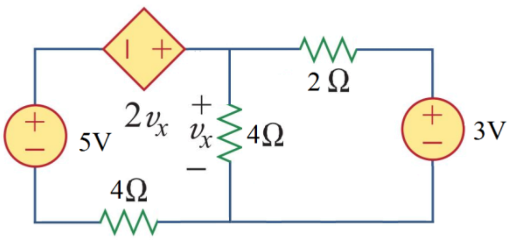
Dependent source

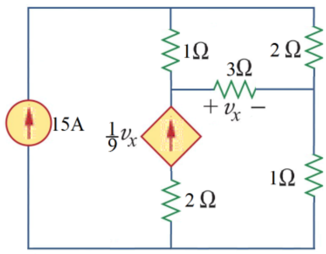
An independent voltage/current source is an idealized circuit component that fixes the voltage or current in a branch, respectively, to a specified value.

Example 13

Determine the currents of the network shown below







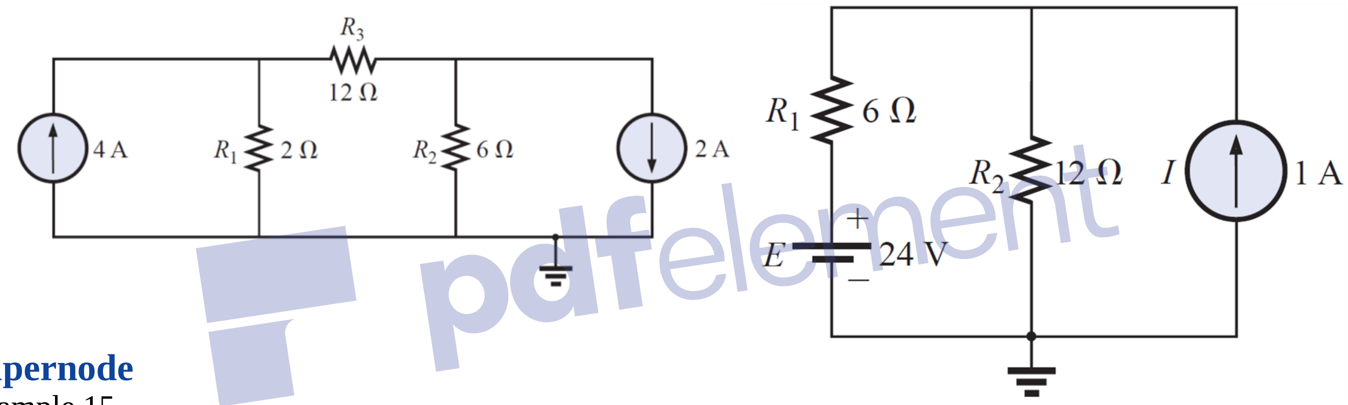
NODAL ANALYSIS

A **node** is defined as a junction of two or more branches. If we now define one node of any network as a reference (that is, a point of zero potential or ground), the remaining nodes of the network will all have a fixed potential relative to this reference. For a network of N nodes, therefore, there will exist (N -1) nodes with a fixed potential relative to the assigned reference node. Equations relating these nodal voltages can be written by applying Kirchhoff's current law at each of the (N -1) nodes. To obtain the complete solution of a network, these nodal voltages are then evaluated in the same manner in which loop currents were found in loop analysis.

The nodal analysis method is applied as follows:

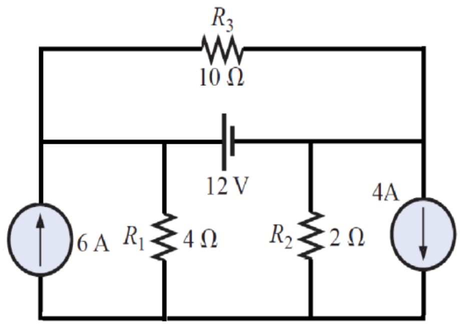
1. Determine the number of nodes within the network.
2. Pick a reference node, and label each remaining node with a subscripted value of voltage: V1, V2, and so on.
3. Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law.
4. Solve the resulting equations for the nodal voltages.

Example 14
Determine the nodal voltages



Supernode

Example 15
Determine the nodal voltages



Delta – star and star – delta convertors

$$R_1 = \frac{R_a R_b}{R_a R_b R_c}$$

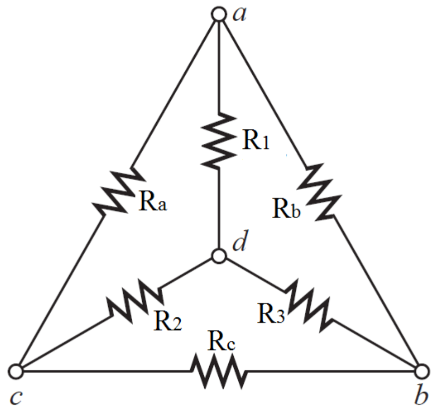
$$R_2 = \frac{R_a R_c}{R_a R_b R_c}$$

$$R_3 = \frac{R_c R_b}{R_a R_b R_c}$$

$$R_a = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

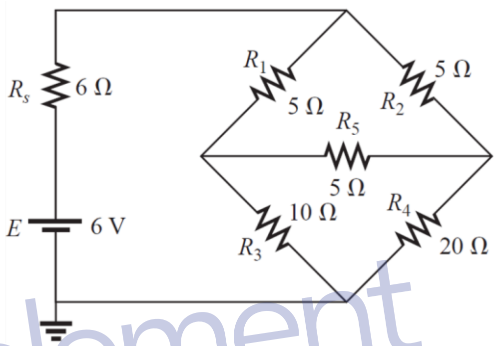
$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$



Example

Calculate the total resistance of the circuit shown below



Capacitors and Inductors

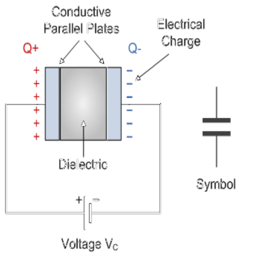
Introduction

Capacitors and inductors are passive elements, each of which has the ability to both store and deliver finite amount of energy. They differ from ideal source in the respect, since they cannot sustain a finite average power flow over an infinite time interval. Although they are classed as linear elements, the current-voltage relationships for these elements are time-dependent, leading to many interesting circuits.

The capacitor

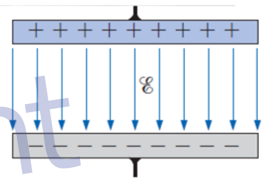
Just like the Resistor, the **Capacitor**, sometimes referred to as a Condenser, is a simple passive device that is used to "store electricity". The capacitor is a component which has the ability or "capacity" to store energy in the form of an electrical charge producing a potential difference (*Static Voltage*) across its plates, much like a small rechargeable battery.

The amount of potential difference present across the capacitor depends upon how much charge was deposited onto the plates by the work being done by the source voltage and also by how much capacitance the capacitor has and this is illustrated below.



Capacitance is the electrical property of a capacitor and is the measure of a capacitors ability to store an electrical charge onto its two plates with the unit of capacitance being the Farad (abbreviated to F) named after the British physicist Michael Faraday.

Capacitance is defined as being that a capacitor has the capacitance of One Farad when a charge of One Coulomb is stored on the plates by a voltage of One volt. Capacitance, C is always positive and has no negative units. However, the Farad is a very large unit of measurement to use on its own so sub-multiples of the Farad are generally used such as micro-farads, nano-farads and pico-farads, for example, the capacitance is determined by



$$C = \frac{Q}{V}$$

C = farads (F)
Q = coulombs (C)
V = volts (V)

If a potential difference of V volts is applied across the two plates separated by a distance of d, the electric field strength between the plates is determined by

$$\mathscr{E} = \frac{V}{d} \quad \text{(volts/meter, V/m)}$$

The ratio of the flux density to the electric field intensity in the dielectric is called the **permittivity** of the dielectric:

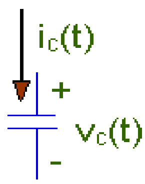
$$\epsilon = \frac{D}{\mathscr{E}} \quad \text{(farads/meter, F/m)}$$

For a vacuum, the value of ϵ (denoted by ϵ_0) is 8.85×10^{-12} F/m. The ratio of the permittivity of any dielectric to that of a vacuum is called the **relative permittivity,** ϵ_r. It simply compares the permittivity of the dielectric to that of air. In equation form,

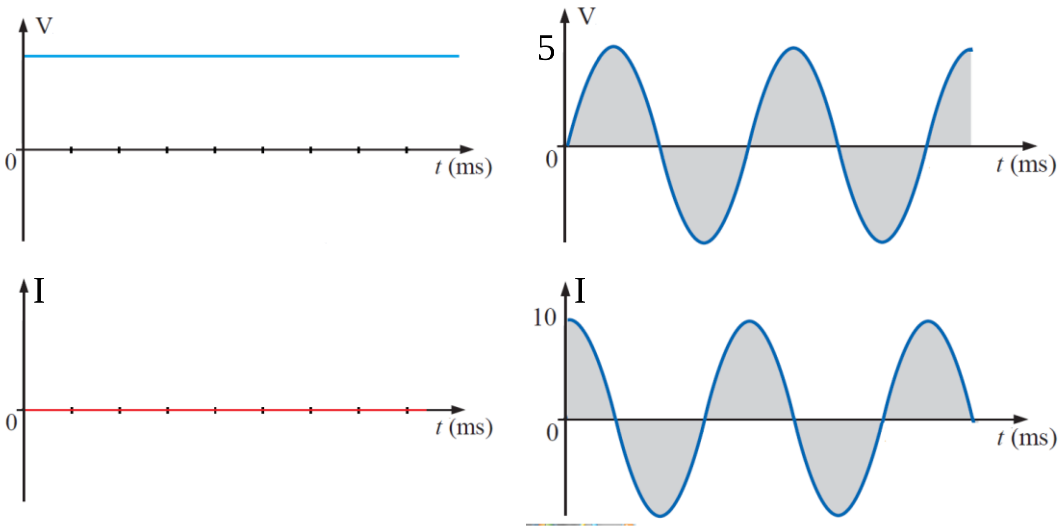
$$\epsilon_r = \frac{\epsilon}{\epsilon_o}$$

The current i_c associated with a capacitance C is related to the voltage across the capacitor by

$$i_c = C\frac{dv_c}{dt}$$



Example:
Determine the current i following through the 2F capacitor for the two waveforms of following figures.



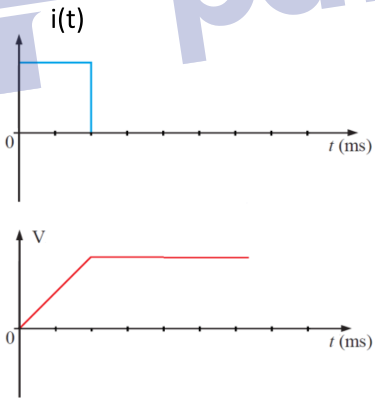
The capacitor voltage may be expressed in terms of the current by integrating i_c. We first obtain

$$dv_c = \frac{1}{C} i(t)dt$$

And then integrate between the times t_0 and t and between the corresponding voltages $v(t_0)$ and $v(t_0)$ as.

$$v = \frac{1}{C}\int_{t_0}^{t} i(\acute{t})d\acute{t} + v(t_0)$$

Example:
Find the capacitor voltage that is associated with the current show graphically in Figure below. The value of the capacitor is 5μF.



$$v_c(t) = \frac{1}{C}\int_{0}^{t} i d\acute{t} + v(t_0)$$

$$v_c(t) = 400t \quad 0 < t < 2$$

$$v_c(t) = 8 \qquad t > 2$$

Energy storage

To determine the energy stored in a capacitor, we begin with the power delivered to it.

$$p = vi = Cv\frac{dv}{dt}$$

The change in the energy stored in its electric field is simply

$$\int_{t_0}^{t} p d\acute{t} = C\int_{t_0}^{t} v\frac{dv}{dt} d\acute{t} = C\int_{v(t_0)}^{v(t)} \acute{v}\, d\acute{v} = \frac{1}{2}C[[v(t)]^2 - [v(t_0)]^2]$$

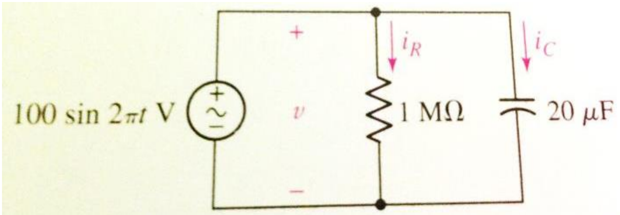
And thus

$$w_c(t) - w_c(t_0) = \frac{1}{2}C[[v(t)]^2 - [v(t_0)]^2]$$

Finally

$$w_c(t) = \frac{1}{2}Cv^2$$

Example:
Find the maximum energy stored in the capacitor of following figure and the energy dissipated in the resistor over the interval $0 < t < 0.5\ s$.

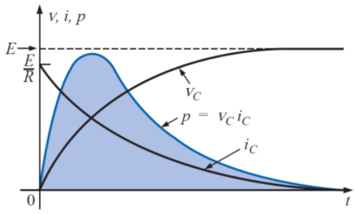


Solution:

$$w_c(t) = \frac{1}{2}Cv^2 = 0.1\sin^2 2\pi t \quad J$$

$$P_R = \frac{v^2}{R} = \frac{10^2\sin^2 2\pi t}{10^6} \quad \text{W}$$

$$w_R = \int_0^{0.5} P_R\, dt = \int_0^{0.5} \frac{10^2\sin^2 2\pi t}{10^6}\, dt \quad \text{J}$$

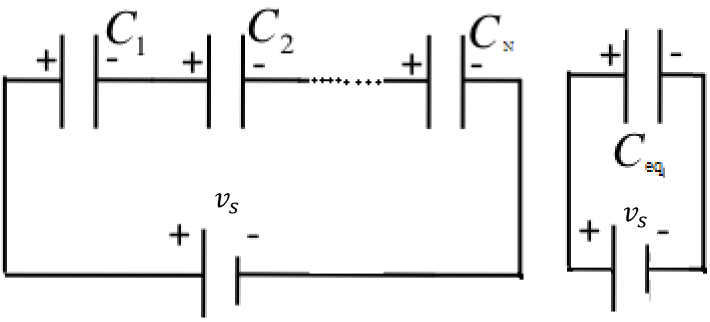


Capacitors in Series

For the following circuit the source voltage can be written as

$$v_s = \sum_{n=1}^{N} v_n = \sum_{n=1}^{N}\left[\frac{1}{C_n}\int_{t_0}^{t} i(\acute{t})d\acute{t} + v(t_0)\right]$$

$$= \sum_{n=1}^{N}\frac{1}{C_n}\int_{t_0}^{t} i(\acute{t})d\acute{t} + \sum_{n=1}^{N} v(t_0)$$



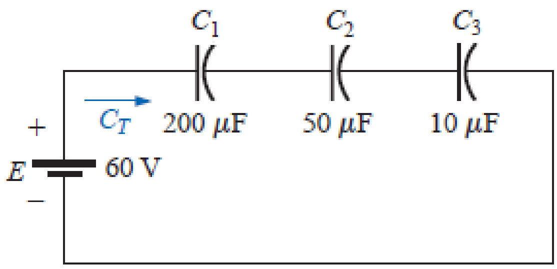
And

$$v_s = \frac{1}{C_{eq}}\int_{t_0}^{t} i(\acute{t})d\acute{t} + \sum_{n=1}^{N} v(t_0)$$

As a result

$$\frac{1}{C_{eq}} = \sum_{n=1}^{N}\frac{1}{C_n} = \frac{1}{C_1} + \frac{1}{C_2} \ldots\ldots\ldots + \frac{1}{C_N}$$

Example:
For the circuit shown below
1- Find the total capacitance.
2- Determine the charge on each plate.
3- Find the voltage across each capacitor.



Solution

1- $\frac{1}{C_T} = \sum_{n=1}^{3}\frac{1}{C_n} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{200\times10^{-6}} + \frac{1}{50\times10^{-6}} + \frac{1}{10\times10^{-6}} = (0.005 + 0.02 + 0.1)\times10^6 = 0.125\times10^6$

$$C_T = \frac{1}{0.125\times10^6} = 8\ \mu F$$

2- $Q_T = Q_1 = Q_2 = Q_3 = C_T E = 8\times10^{-6}\times60 = 480\mu C$

3- $v_1 = \frac{Q_1}{C_1} = \frac{480\times10^{-6}}{200\times10^{-6}} = 2.4\ V$

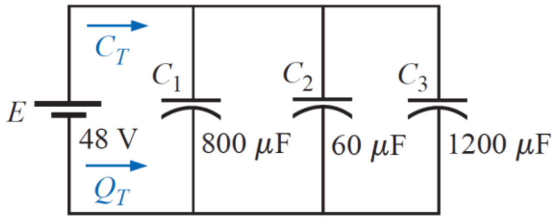
$v_2 = \frac{Q_2}{C_2} = \frac{480\times10^{-6}}{50\times10^{-6}} = 9.6\ V$

$v_3 = \frac{Q_3}{C_3} = \frac{480\times10^{-6}}{10\times10^{-6}} = 48\ V$

Example:

For the network of Figure shown below

a. Find the total capacitance.
b. Determine the charge on each plate.
c. Find the total charge.



Solution

a- $C_T = C_1 + C_2 + C_3 = 800 \times 10^{-6} + 60 \times 10^{-6} + 1200 \times 10^{-6} = 2060\ \mu F$

b- $Q_1 = C_1 E = 800 \times 10^{-6} \times 48 = 38.4\ mC$

$Q_2 = C_2 E = 60 \times 10^{-6} \times 48 = 2.88\ mC$

$Q_3 = C_3 E = 1200 \times 10^{-6} \times 48 = 57.6\ mC$

c- $Q_T = Q_1 + Q_2 + Q_3 = 38.4 \times 10^{-3} + 2.88 \times 10^{-3} + 57.6 \times 10^{-3} = 98.88\ mC$

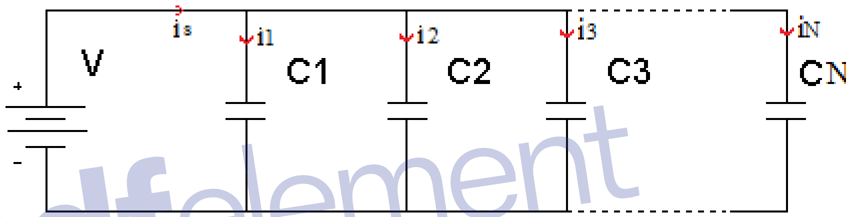
Capacitors in parallel

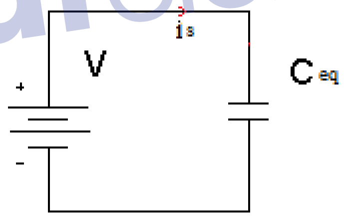
The circuits of following figure enable us to establish the value of capacitor which is equivalent to N parallel capacitors are

$$i_s = \sum_{n=1}^{N} i_n = \sum_{n=1}^{N} C_n \frac{dv}{dt}$$

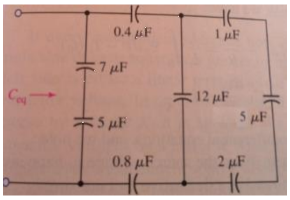
$$= C_{eq} \frac{dv}{dt}$$

$$C_{eq} = \sum_{n=1}^{N} C_n = C_1 + C_2 + \cdots + C_N$$





Example:
Find C_{eq} for the following network



$C_1 = \frac{1\times5\times2}{1\times5+1\times2+2\times5} = \frac{10}{17}\mu F$
$C_2 = \frac{10}{17} + 12 = 12.588\mu F$
$C_3 = \frac{12.588\times0.4\times0.8}{12.588\times0.4+0.4\times0.8+12.588\times0.8} =$
$C_4 = \frac{7\times5}{7+5} = \frac{35}{12}\mu F$
$C_{eq} = C_3 + C_4 =$

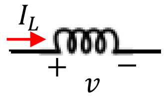
Important Characteristics of an Ideal Capacitor

1- There is no current through a capacitor if the voltage across it is not change with time. A capacitor is an open circuit to d.c.
2- A finite amount of energy can be stored in the capacitor even if the current trough the capacitor is zero, such as when the voltage across it is constant.
3- It is impossible to change the voltage across the capacitor by a finite amount in zero time, as this requires an infinite current through the capacitor.
4- A capacitor never dissipate energy, but only stores it. Although this is true for the mathematical model, it is not true for a physical capacitor due to finite resistor associated with the dielectric as well as the packaging.

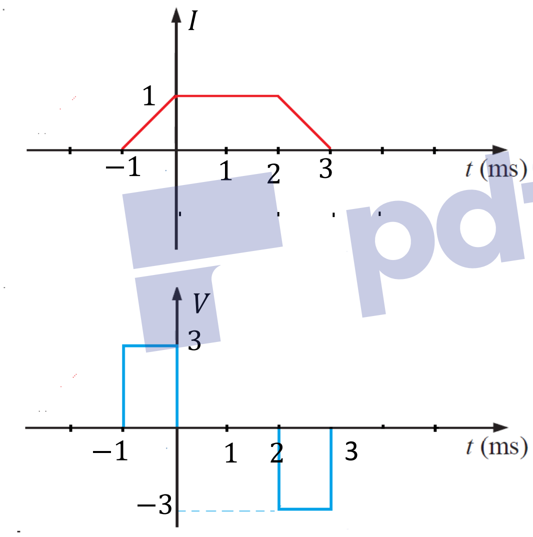
The Inductor

In the early 1800s the Danish science Oersted showed that a current carrying conductor produced a magnetic field. Shortly thereafter, Ampers made some careful measurements which demonstrated that this magnetic field was linearly related to the current which produced it. The English experimental Michael faraday and the American inventor Joseph Henry discovered that a changing magnetic field could induce a voltage in a neighboring circuit. They showed that this voltage was proportional to the time rate of change of the current producing the magnetic field. Mathematically can be expressed as

$$v = L\frac{di}{dt}$$



Example:
Given the waveform of the current in a 3H inductor as shown in figure below, determine the inductor voltage and sketch it.



$$V = L\frac{di}{dt}$$

$$V = 3 \times 1 = 3 \qquad -1 < t < 0$$

$$V = 0 \qquad 0 < t < 2$$

$$V = 3 \times (-1) = -3 \quad 2 < t < 3$$

To calculate the inductor current, rewrite the voltage expression as

$$di = \frac{1}{L}vdt$$

$$\int_{i(t_0)}^{i(t)} d\acute{i} = \frac{1}{L}\int_{t_0}^{t} vd\acute{t}$$

$$i(t) - i(t_0) = \frac{1}{L}\int_{t_0}^{t} vd\acute{t}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} vd\acute{t} + i(t_0)$$

Example:
The voltage across a 2H inductor is known to be $6cos5t\ V$. Determine the resulting inductor current if $i\left(t = -\frac{\pi}{2}\right) = 1A$.
Example:
A 100 mH inductor has voltage $v_L = 2e^{-3t}V$ across its terminals. Determine the resulting inductor current if $i_L(-0.5) = 1A$.

The observed power is given by the current-voltage product

$$p = vi = Li\frac{di}{dt}$$

The energy w_L accepted by the inductor is stored in the magnetic field around the coil. The change in tis energy is expressed by the integral of the power over the desired time interval:

$$\int_{t_0}^{t} p d\acute{t} = L\int_{t_0}^{t} i\frac{di}{d\acute{t}}d\acute{t} = L\int_{i(t_0)}^{i(t)} \acute{i}d\acute{i} = \frac{1}{2}L[[i(t)]^2 - [i(t_0)]^2]$$

Thus

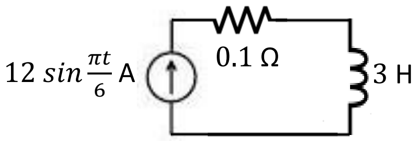
$$w_L(t) - w_L(t_0) = \frac{1}{2}L[[i(t)]^2 - [i(t_0)]^2]$$

When $t_0 = -\infty$ and $i(t_0) = 0$, so that the energy can be expressed as

$$w_L(t) = \frac{1}{2}LI^2$$

Example:

Find the maximum energy stored in the inductor of following figure and calculate how much energy is dissipated in the resistor in the time during which the energy is being stored in, and then recovered from, the inductor.



Solution

The energy stored in the inductor is

$w_L(t) = \frac{1}{2}Li^2 = 216sin^2\frac{\pi t}{6}$ J

At $t = 0, w_L = 0$

At $t = 3\ s, w_L = 216$ J

The power dissipated in the resistor is

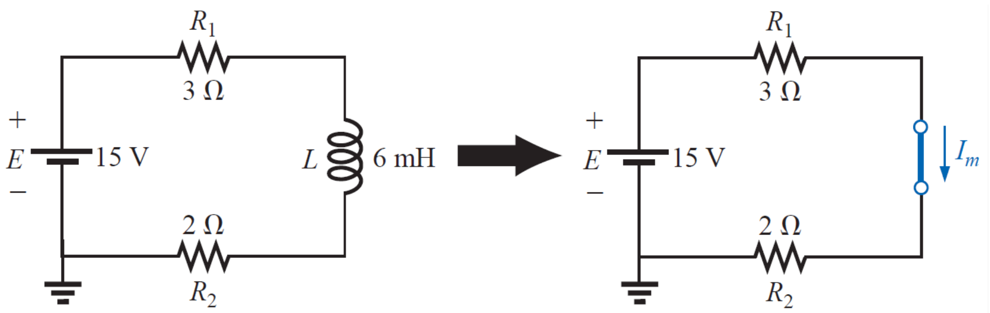
$p_R = i^2R = 14.4sin^2\frac{\pi t}{6}$ w

The energy converted into heat in the resistor within 6 s interval is

$w_R = \int_0^6 14.4sin^2\frac{\pi t}{6}dt = \int_0^6 \frac{14.4}{2}\left(1 - cos\frac{\pi}{3}t\right)dt = 43.2\ J$

Example:

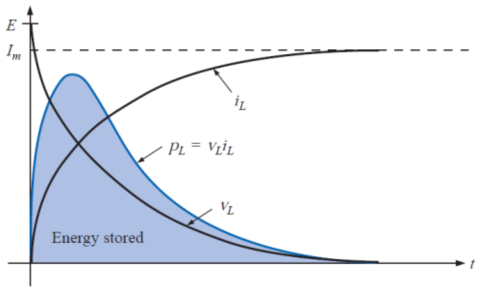
Find the energy stored by the inductor in the circuit shown below when the current through it has reached its final value.



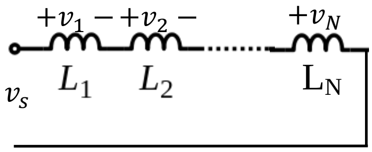
Solution

$$I_m = \frac{E}{R_1 + R_2} = \frac{15}{3+2} = 3\ A$$

$$w_L(t) = \frac{1}{2}Li^2 = \frac{1}{2} \times 6 \times 10^{-3} \times 3^2 = 27\ mJ$$



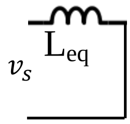
Inductors in series



$$v_s = v_1 + v_2 + \cdots..+v_N$$

$$= L_1\frac{di}{dt} + L_2\frac{di}{dt} + \cdots..+L_N\frac{di}{dt}$$

$$= (L_1 + L_2 + \cdots..+L_N)\frac{di}{dt}$$

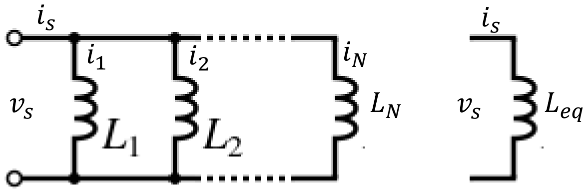


$$v_s = L_{eq}\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + \cdots..+L_N$$

Inductors in Parallel

The combination of a number of parallel inductors is accomplished by writing the single nodal equation for the original circuit



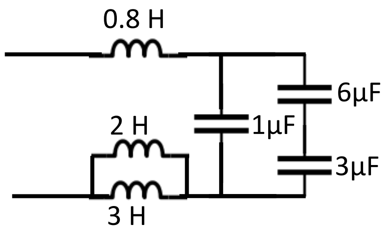
$$i_s = \sum_{n=1}^{N} i_n = \sum_{n=1}^{N}\left[\frac{1}{L_n}\int_{t_0}^{t} v(\acute{t})d\acute{t} + i_n(t_0)\right]$$

$$= \sum_{n=1}^{N}\frac{1}{L_n}\int_{t_0}^{t} v(\acute{t})d\acute{t} + \sum_{n=1}^{N} i_n(t_0)$$

$$i_s = \frac{1}{L_{eq}}\int_{t_0}^{t} v(\acute{t})d\acute{t} + i_s(t_0)$$

$$\frac{1}{L_{eq}} = \sum_{n=1}^{N}\frac{1}{L_n} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}$$

Example:
Simplify the network using series-parallel combination.



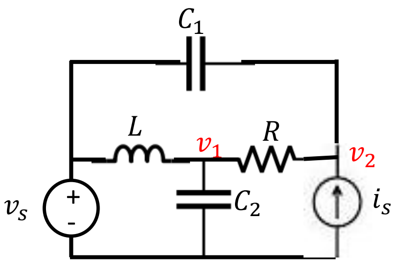
$$C_{eq} = \frac{6\times 3}{9} + 1 = 3\mu F$$

$$L_{eq} = \frac{2\times 3}{5} + 0.8 = 2H$$

Important Characteristics of an Ideal Capacitor

1- There is no voltage across an inductor if the current through it is not changing wit time. An inductor is therefore a short circuit to dc.
2- A finite amount of energy can be stored in an inductor even if the voltage across the inductor is zero, such as when the current through it is constant.
3- It is impossible to change the current through an inductor by a finite amount in zero time, for this requires an infinite voltage across the inductor.
4- The inductor never dissipates energy, but only stories it. Although this is true for the mathematical model, it is not true for a physical inductor due to series resistance.

Example:
Write appropriate nodal equations for the following figure.



At node 1

$$\frac{1}{L}\int_{t_0}^{t}(v_1 - v_s)dt' + i_L(t_0) + \frac{v_1 - v_2}{R} + C_2\frac{dv_1}{dt} = 0$$

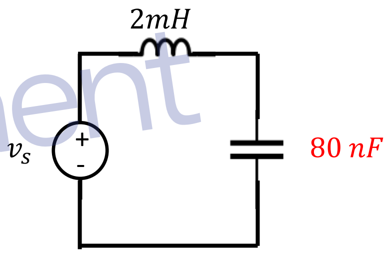
$$\frac{v_1}{R} + C_2\frac{dv_1}{dt}\ \frac{1}{L}\int_{t_0}^{t} v_1 dt' - \frac{v_2}{R} = \frac{1}{L}\int_{t_0}^{t} v_s dt' - i_L(t_0)$$

At node 2

$$C_1\frac{d(v_2 - v_s)}{dt} + \frac{v_2 - v_1}{R} + i_s = 0$$

$$-\frac{v_1}{R} + \frac{v_2}{R} + C_1\frac{dv_2}{dt} = C_1\frac{dv_s}{dt} + i_s$$

Example:
Determine v_s for the circuit shown below if $v_c(t) = 4\cos 10^5 t$



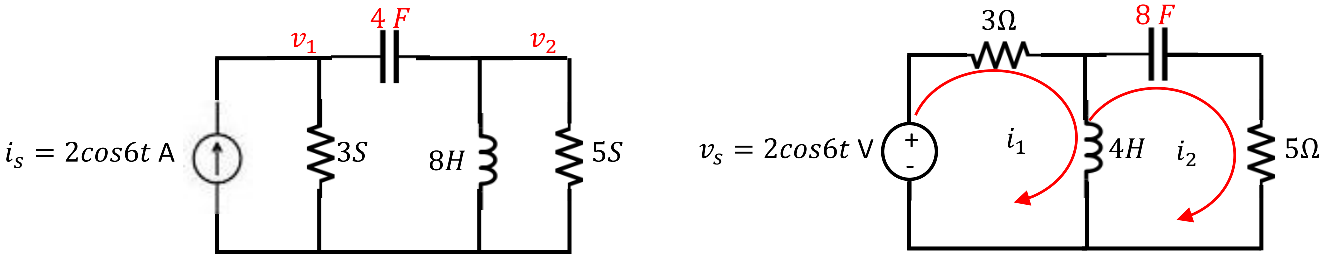
$$v_s(t) = v_L(t) + v_C(t)$$

$$i_c = C\frac{dv_C(t)}{dt} = -80 \times 10^{-9} \times 4 \times 10^5 sin10^5 t = -320 \times 10^{-4} sin10^5 t$$

$$v_L(t) = L\frac{di}{dt} == -320 \times 10^{-4} \times 2 \times 10^{-3} \times 10^5 cos10^5 t = -6.4cos10^5 t$$

$$v_s(t) = 4\cos 10^5 t - 6.4cos10^5 t = -2.4cos10^5 t \ \ \text{V}$$

Duality



Two mesh currents can be seen in the figure, and the mesh equations are

$$3i_1 + 4\frac{di_1}{dt} - 4\frac{di_2}{dt} = 2cos6t$$

$$-4\frac{di_1}{dt} + 4\frac{di_2}{dt} + \frac{1}{8}\int_0^t i_2 d\acute{t} + 5i_2 = 0$$

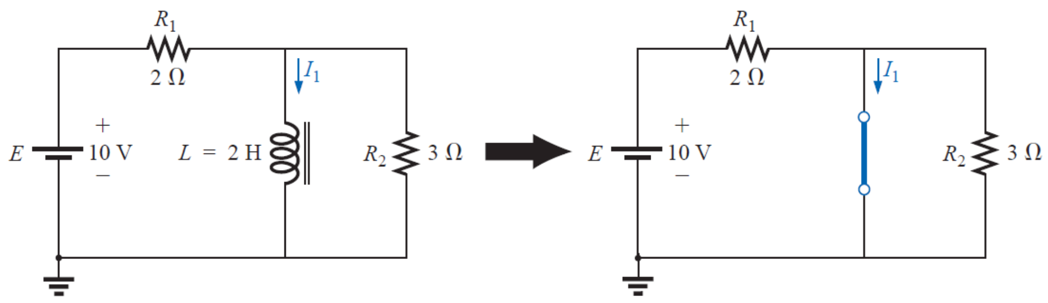
We may now construct the two equations that describe the exact dual of our circuit. We which these to be nodal equations, and thus begin by replacing the mesh currents by the two nodal voltages. We obtain

$$3v_1 + 4\frac{dv_1}{dt} - 4\frac{dv_2}{dt} = 2cos6t$$

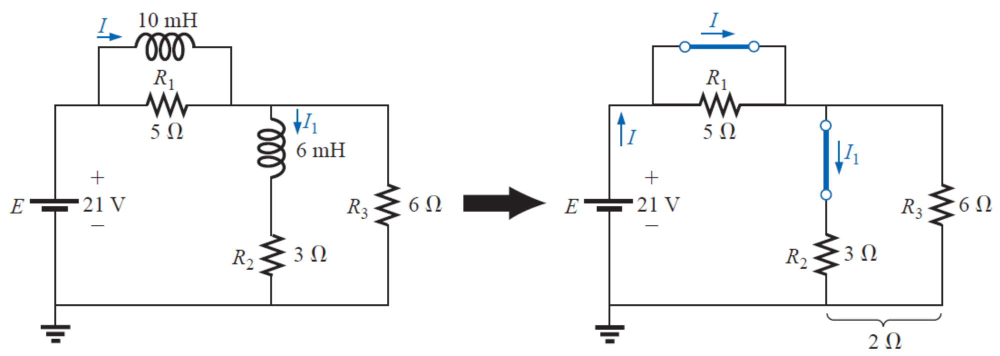
$$-4\frac{dv_1}{dt} + 4\frac{dv_2}{dt} + \frac{1}{8}\int_0^t v_2 d\acute{t} + 5v_2 = 0$$

Basic RL and RC Circuits

The RL circuit with D.C (steady state)
The inductor is short time at $t = \infty$
Calculate the inductor current for circuits shown below.



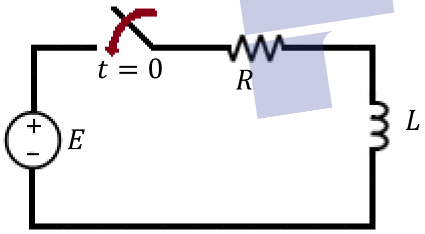
$$I_L = \frac{E}{R_1} = \frac{10}{2} = 5A$$



$$I_{L_1} = \frac{E}{\left(\frac{R_2 R_3}{R_2 + R_3}\right)}$$

$$I_{L_2} = I_{L_1} \frac{R_3}{R_2 + R_3}$$

R-L TRANSIENTS: STORAGE CYCLE



$$-E + Ri + L\frac{di}{dt} = 0$$

$$Ri + L\frac{di}{dt} = E$$

$$L\frac{di}{dt} = E - Ri$$

$$Ldi = (E - Ri)dt$$

$$\frac{Ldi}{(E - Ri)} = dt$$

$$\int \frac{Ldi}{(E - Ri)} = \int dt$$

$$-\frac{L}{R} ln(E - Ri) = t + k$$

at $t = 0, i = 0$, therefore

$$-\frac{L}{R} ln(E) = k$$

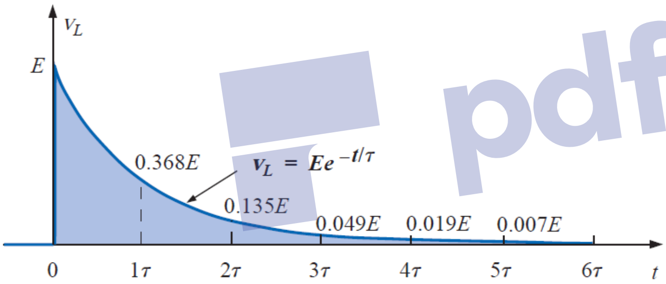
And

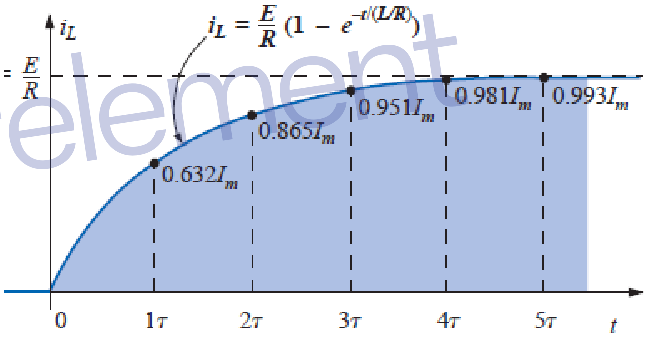
$$\frac{L}{R}ln(E-Ri)=t-\frac{L}{R}ln(E)$$
$$-\frac{L}{R}ln(E-Ri)+\frac{L}{R}ln(E)=t$$
$$-\frac{L}{R}\left(ln\left(\frac{E-Ri}{E}\right)\right)=t$$
$$\frac{E-Ri}{E}=e^{-\frac{R}{L}t}$$
$$i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)=\frac{E}{R}\left(1-e^{-\frac{t}{\tau}}\right)$$

$$\boxed{\tau=\frac{L}{R}}\qquad \text{(seconds, s)}$$

$$\boxed{i_L=I_m(1-e^{-t/\tau})=\frac{E}{R}(1-e^{-t/(L/R)})}$$

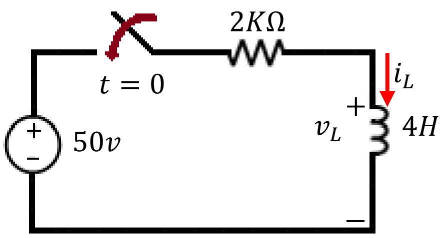
$$\boxed{V_L=Ee^{-t/\tau}}$$





Example:

Find the mathematical expression for the transient behaviour of i_L and v_L.



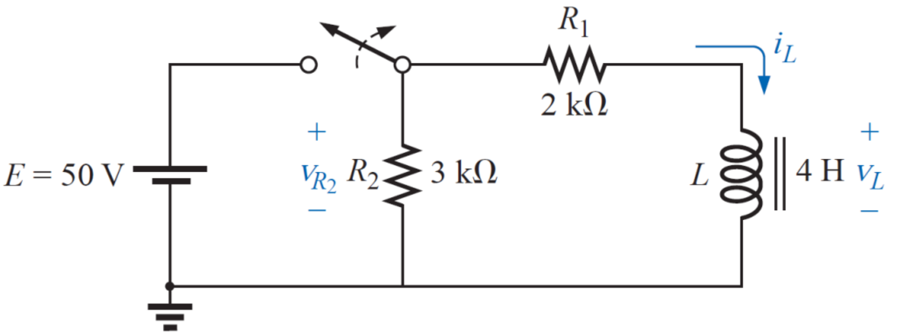
$\tau=\frac{L}{R}=\frac{4}{2\times10^3}=2ms$

$i_L=\frac{E}{R}\left(1-e^{-\frac{t}{\tau}}\right)=\frac{50}{2\times10^3}(1-e^{-500t})=25(1-e^{-500t})mA$

$v_L=Ee^{-\frac{t}{\tau}}=50e^{-500t}\,V$

Example:

For the circuit shown below, calculate the mathematical expression of $i_L, v_L, v_{R_1}, v_{R_2}$ before and after the storage phase has been complete ant the switch is open.



1-switch on

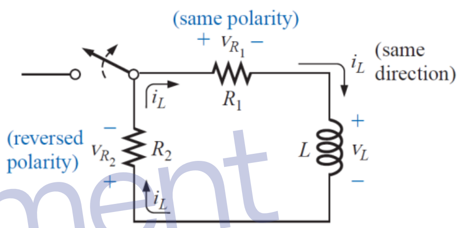
$\tau = \frac{L}{R_{eq}} = \frac{4}{2\times10^3} = 2ms$

$i_L = \frac{E}{R_1}\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{50}{2\times10^3}(1 - e^{-500t}) = 25(1 - e^{-500t})mA$

$v_L = Ee^{-\frac{t}{\tau}} = 50e^{-500t}\ V$

$v_{R_1} = i_L R_1 = \frac{E}{R_1}R_1\left(1 - e^{-\frac{t}{\tau}}\right) = 50(1 - e^{-500t})v$

$v_{R_2} = E = 50\ V$



2-switch off

After the storage phase has passed and steady-state conditions are established, the switch can be opened without the sparking effect or rapid discharge due to the resistor R_2, which provides a complete path for the current i_L. The voltage v across the inductor will reverse polarity and have a magnitude determined by

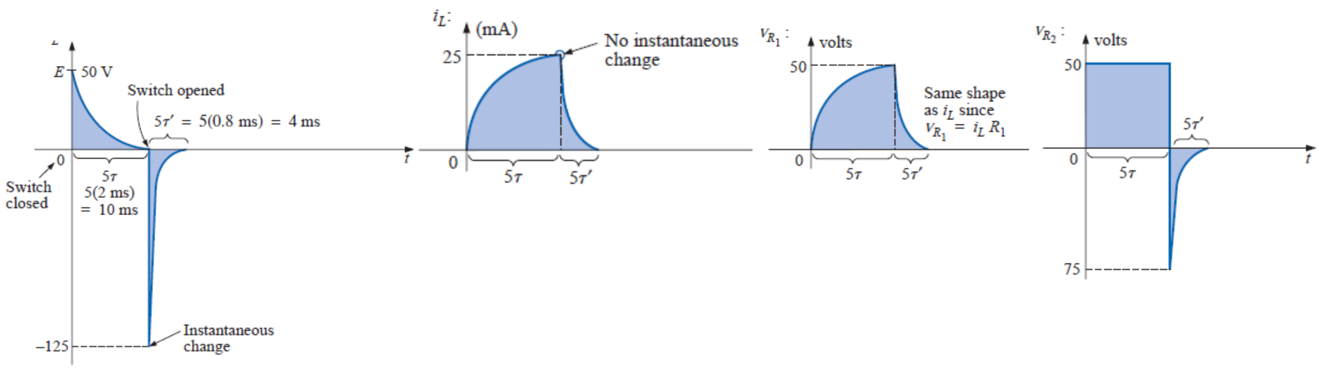
$\acute{\tau} = \frac{L}{R_{eq}} = \frac{4}{2\times10^3+3\times10^3} = 0.8\ ms$

$i_L = \frac{E}{R_1}e^{-\frac{t}{\acute{\tau}}} = \frac{50}{2\times10^3}e^{-\frac{t}{\acute{\tau}}} = 25e^{-\frac{t}{0.8\times10^{-3}}}\ mA$

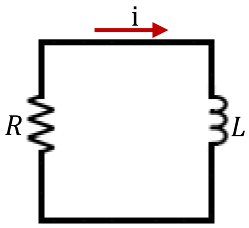
$v_L = -i_L(R_1 + R_2) = -\frac{E}{R_1}(R_1 + R_2)e^{-\frac{t}{\acute{\tau}}} = -E\left(1 + \frac{R_2}{R_1}\right)e^{-\frac{t}{0.8\times10^{-3}}} = -50\left(1 + \frac{3}{2}\right)e^{-\frac{t}{0.8\times10^{-3}}} = -75e^{-\frac{t}{0.8\times10^{-3}}}V$

$v_{R_1} = i_L R_1 = \frac{E}{R_1}R_1 e^{-\frac{t}{\acute{\tau}}} = 50e^{-\frac{t}{0.8\times10^{-3}}t}\ v$

$v_{R_2} = -i_L R_2 = -\frac{E}{R_1}R_2 e^{-\frac{t}{\acute{\tau}}} = -\frac{50}{2}3e^{-\frac{t}{0.8\times10^{-3}}t} = -75e^{-\frac{t}{0.8\times10^{-3}}t}\ V$



The Source Free RL circuit



Using KVL, leads

$$Ri + L\frac{di}{dt} = 0$$

This equation represents a differential equation and can be solved by several different methods

$$\frac{di}{i} = -\frac{R}{L}dt$$

Since the current is I_0 at $t = 0$ and $i(t)$ at time t, we may equate the two definite integrals with are obtained by integrating each side between the corresponding limits

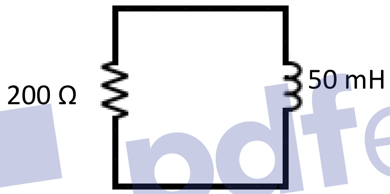
$$\int_{I_0}^{i(t)} \frac{di'}{i} = -\frac{R}{L}\int_0^t dt'$$

$$i(t) = I_0\, e^{-\frac{R}{L}t}$$

$$v(t) = -v_L\, e^{-\frac{R}{L}t}$$

Example:

If the inductor has a current 2A at t=0, find an expression for $i_L(t)$ valid for $t > 0$, and its value at t=200µs.



$i(t) = I_0\, e^{-\frac{R}{L}t} = 2e^{-\frac{200}{50\times10^{-3}}t} = 2e^{-4000t}$A

At t=200µs

$i(200\mu s) = 2e^{-4000\times200\times10^{-6}} = 2e^{-0.8} = 898.7$ mA

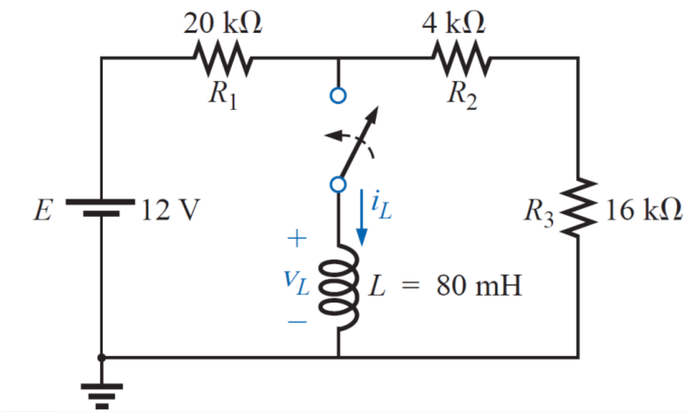
THÉVENIN EQUIVALENT:

Example:

For the network of Figure below

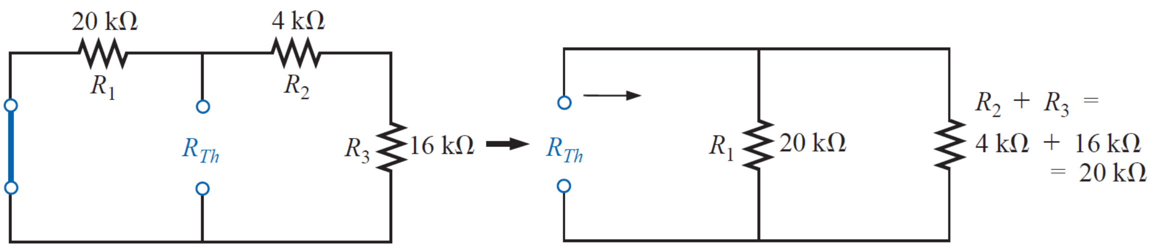
a. Find the mathematical expression for the transient behavior of the current *iL* and the voltage *vL* after the closing of the switch (*Ii* =0 mA).

b. Draw the resultant waveform for each.



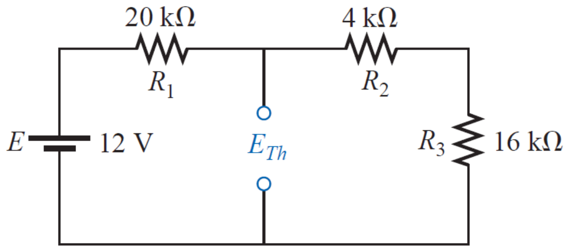
Solutions:

a. Applying Thévenin's theorem to the 80-mH inductor ,yields



$$R_{th} = \frac{(4+16)\times 20}{(4+16)+20} = 10\ K\Omega$$

Applying the voltage divider rule to determine Thevinen voltage



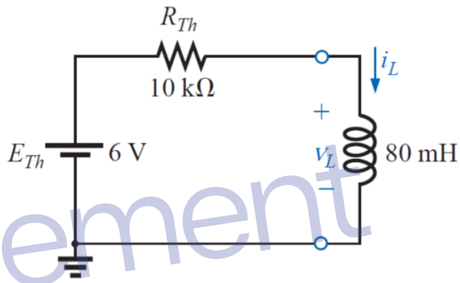
$$E_{th} = 12\frac{(4+16)}{(4+16)+20} = 6\ V$$

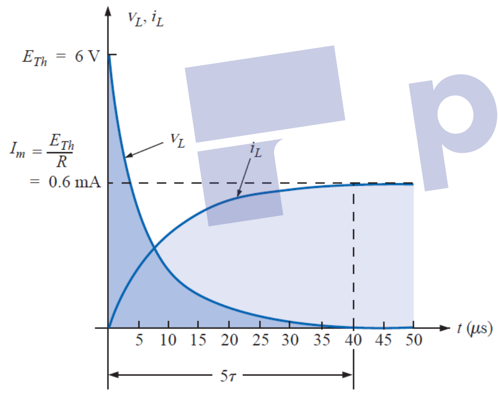
$$\tau = \frac{L}{R_{th}} = \frac{80\times10^{-3}}{10\times10^{3}} = 8\ \mu s$$

$$i_L = \frac{E_{th}}{R_{th}}\left(1-e^{-\frac{t}{\tau}}\right) = \frac{6}{10\times10^3}\left(1-e^{-\frac{t}{8\times10^{-6}}}\right)$$

$$i_L = 0.6\left(1-e^{-\frac{t}{8\times10^{-6}}}\right)mA$$

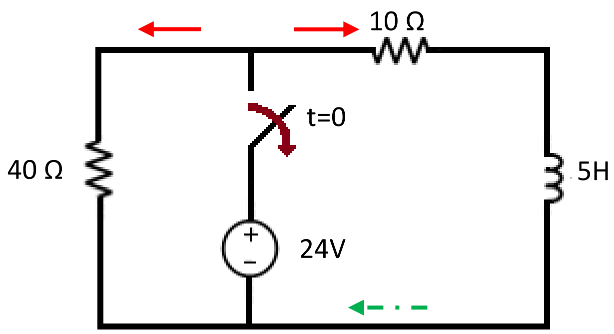
$$v_L = E_{th}e^{-\frac{t}{\tau}} = 6\,e^{-\frac{t}{8\times10^{-6}}}\ V$$





Example:

Find the voltage across 40Ω resistor at t=200ms.



$$I_l = \frac{24}{10} = 2.4\ A$$

$$\tau = \frac{L}{R} = \frac{5}{10} = 0.5\ s$$

At t=0

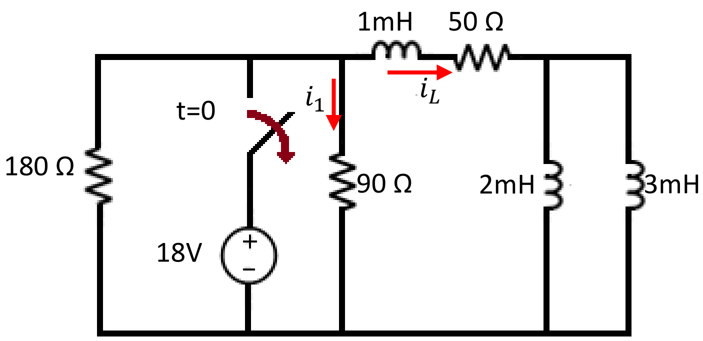
$$\tau = \frac{L}{R_{eq}} = \frac{5}{10+40} = 0.1\ s$$

$$i(t) = I_0\, e^{-\frac{R}{L}t} = 2.4\, e^{-10t}$$

$$V_{40} = i(t)R = -2.4 \times 40 \times e^{-10t} = -96e^{-10t}\ V$$

Example:

Determine both i_1 and i_L for $t > 0$.



$$L_{eq} = \frac{2 \times 3}{2+3} + 1 = 2.2mH$$

$$R_{eq} = \frac{180 \times 90}{180+90} + 50 = 110\Omega$$

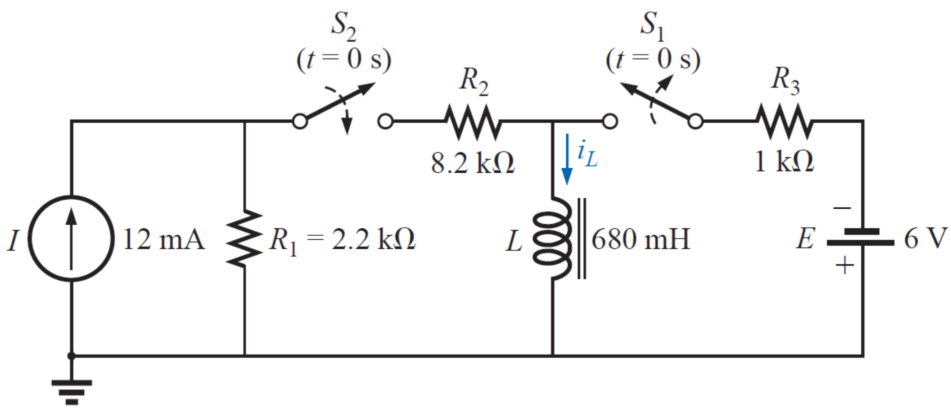
$$I_L = \frac{18}{50} = 360\ mA$$

$$I_L(t) = 360\ e^{-\frac{R_{eq}}{L_{eq}}t} = 360\ e^{-\frac{110}{2.2\times10^{-3}}t} = 360\ e^{-50000t} mA$$

$$I_1(t) = -360 \frac{180}{180+90}\ e^{-50000t} = -240\ e^{-50000t}\ mA$$

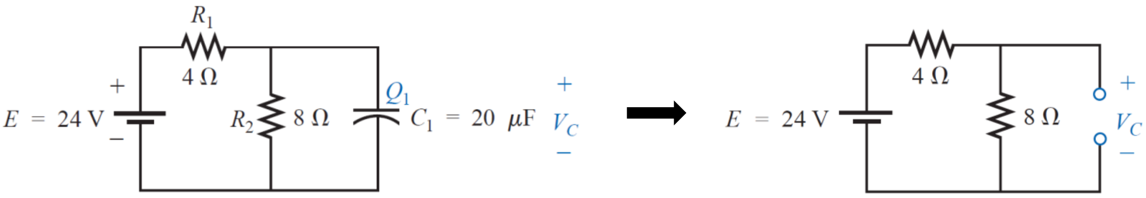
H.W

The switch $S1$ of following Figure has been closed for a long time. At $t = 0$ s, $S1$ is opened at the same instant $S2$ is closed to avoid an interruption in current through the coil.
a. Find the initial current through the coil. Pay particular attention to its direction.
b. Find the mathematical expression for the current i_L following the closing of the switch $S2$.
c. Sketch the waveform for i_L.



RC Circuits

Find the voltage across and charge on capacitor C_1 of Figure below after it has charged up to its final value.



Solution :

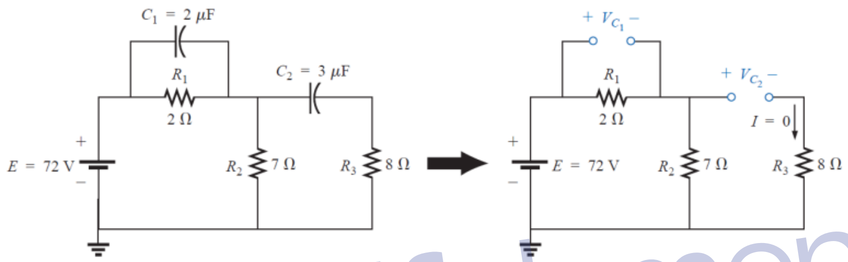
the capacitor is effectively an open circuit for dc after charging up to its final value

$$V_C = E\frac{R_2}{R_1 + R_2} = 24\frac{8}{12} = 16V$$

$$Q_1 = V_C C_1 = 16 \times 20 \times 10^{-6} = 320\ \mu C$$

Example:

Find the voltage across and charge on each capacitor of the network of Figure below after each has charged up to its final value.



Solution

$$V_{C_1} = E\frac{R_1}{R_1 + R_2} = 72\frac{2}{9} = 16V$$

$$V_{C_2} = E\frac{R_2}{R_1 + R_2} = 72\frac{7}{9} = 56V$$

$$Q_1 = V_{C_1} C_1 = 16 \times 2 \times 10^{-6} = 32\ \mu C$$

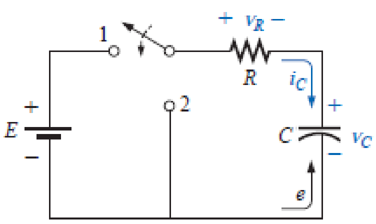
$$Q_2 = V_{C_2} C_2 = 56 \times 3 \times 10^{-6} = 168\ \mu C$$

ENERGY STORED BY A CAPACITOR

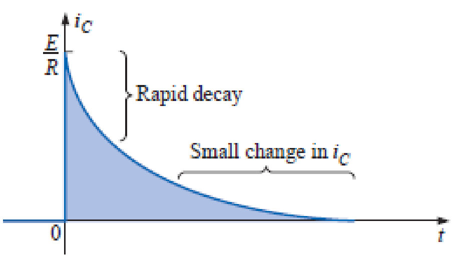
The ideal capacitor does not dissipate any of the energy supplied to it. It stores the energy in the form of an electric field between the conducting surfaces.

$$W_C = \frac{1}{2}CV^2 \quad \text{(J)}$$

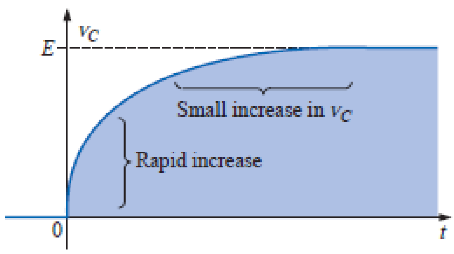
TRANSIENTS IN CAPACITIVE NETWORKS: CHARGING PHASE



Basic charging network.



i_C during the charging phase.



v_C during the charging phase.

$$-E + I_R R + V_C = 0$$
$$-E + RC\frac{dV_C}{dt} + V_C = 0$$
$$RC\frac{dV_C}{dt} = E - V_C$$
$$RC\frac{dV_C}{(E - V_C)} = dt$$
$$\int RC\frac{dV_C}{(E - V_C)} = \int dt$$
$$-RCln(E - V_C) = t + K$$

At t=0, $V_C = 0$, therefore

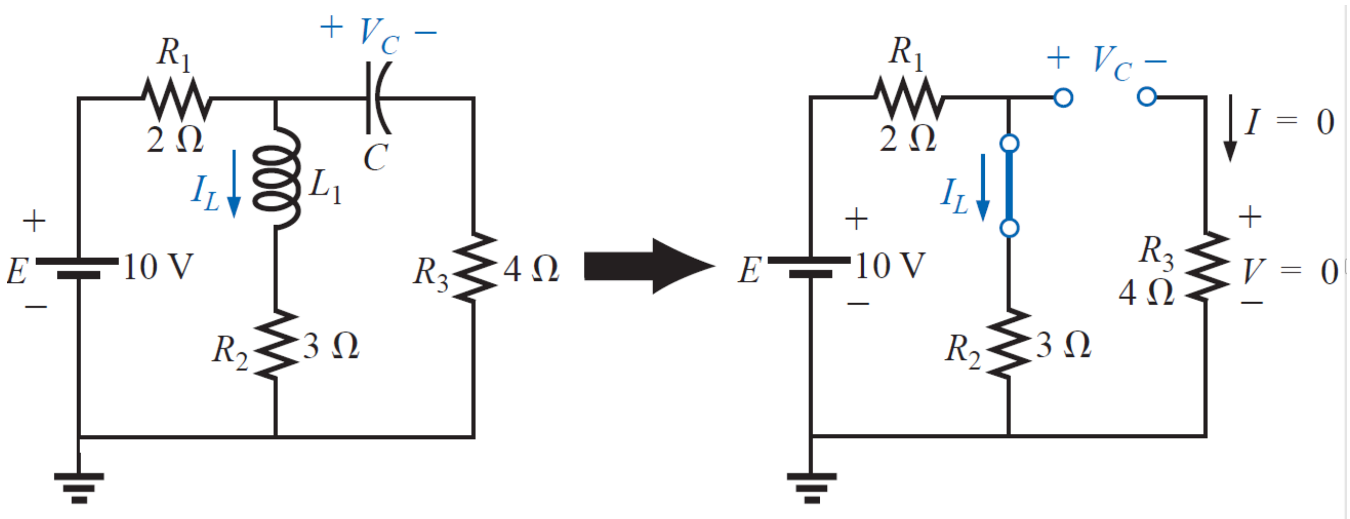
$$-RCln(E) = K$$
$$-RCln(E - V_C) = t - RCln(E)$$
$$-RCln(E - V_C) + RCln(E) = t$$
$$-RCln\left(\frac{E - V_C}{E}\right) = t$$
$$ln\left(\frac{E - V_C}{E}\right) = -\frac{t}{RC}$$
$$\frac{E - V_C}{E} = e^{-\frac{t}{RC}}$$
$$V_C = E - Ee^{-\frac{t}{RC}} = E\left(1 - e^{-\frac{t}{RC}}\right)$$
$$\tau = RC$$

And

$$I_c = I_C = I_0 e^{-\frac{t}{RC}} = \frac{E}{R}e^{-\frac{t}{RC}}$$

RLC Circuits

Example:
Find the current I_L and the voltage V_C for the network



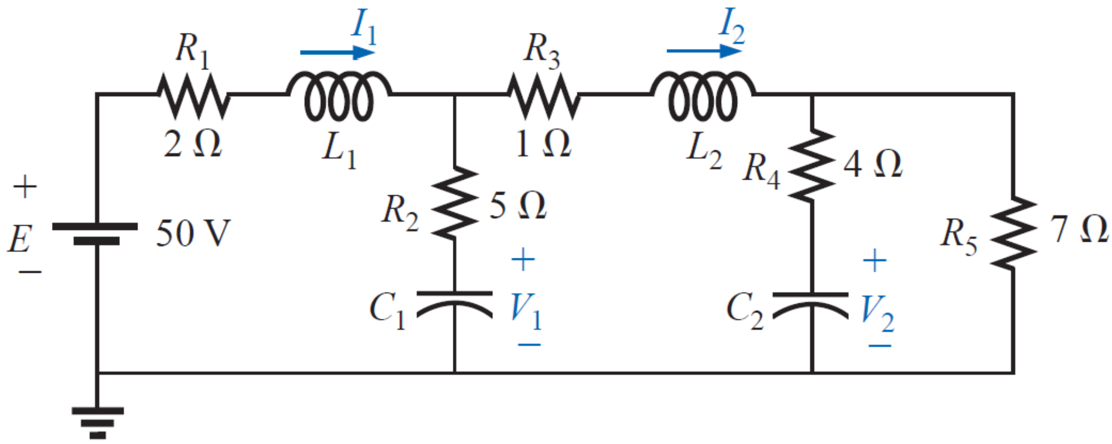
Solution:

$$I_L = \frac{E}{R_1 + R_2} = \frac{10}{5} = 2\,A$$

$$V_L = E\frac{R_2}{R_1 + R_2} = 10\frac{3}{5} = 6\ V$$

H.W

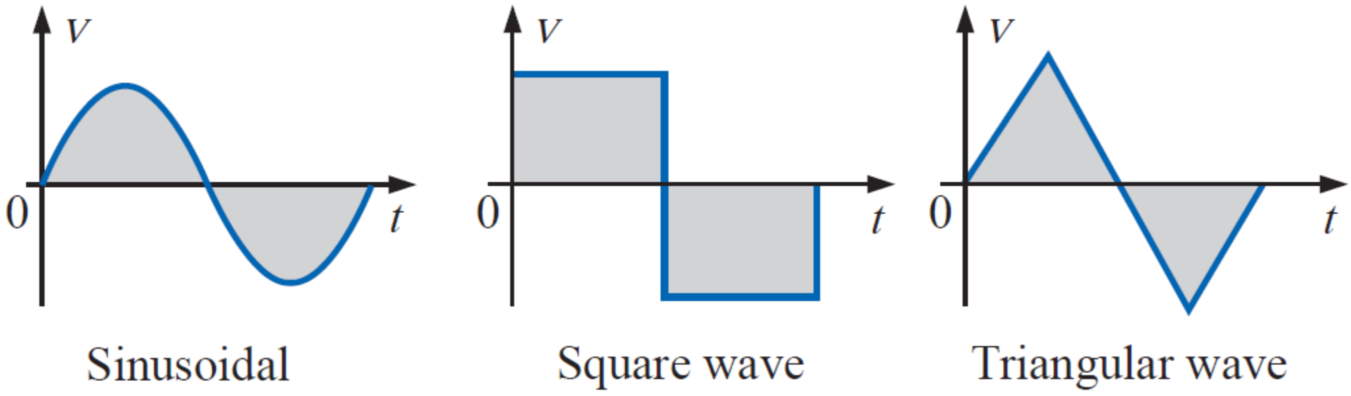
Find the currents $I1$ and $I2$ and the voltages V_1 and V_2 for the network



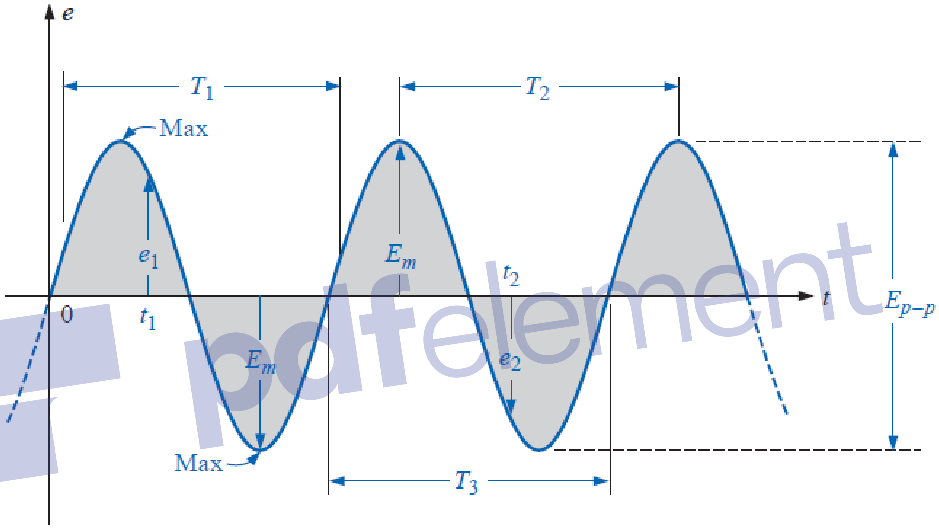
Sinusoidal Alternating Waveforms

Introduction

The term *alternating* indicates only that the waveform alternates between two prescribed levels in a set time sequence



The vertical scaling of the sinusoidal waveform is in volts or amperes and the horizontal scaling is *always* in units of time and can be represented as.



Instantaneous value, Peak amplitude, Peak value, Peak-to-peak value and Period.

Frequency (f)**:** The number of cycles that occur in 1 s.

1 hertz (Hz) = 1 cycle per second (c/s)

$$f = \frac{1}{T}$$

Example:

Find the period of a periodic waveform with a frequency of

a. 60 Hz.

b. 1000 Hz.

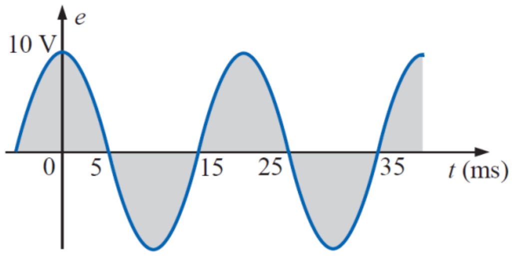
Solution

a- $T = \frac{1}{f} = \frac{1}{60} = 16.67\ ms$

b- $T = \frac{1}{f} = \frac{1}{1000} = 1ms$

Example:

Determine the frequency of the waveform of following Fig



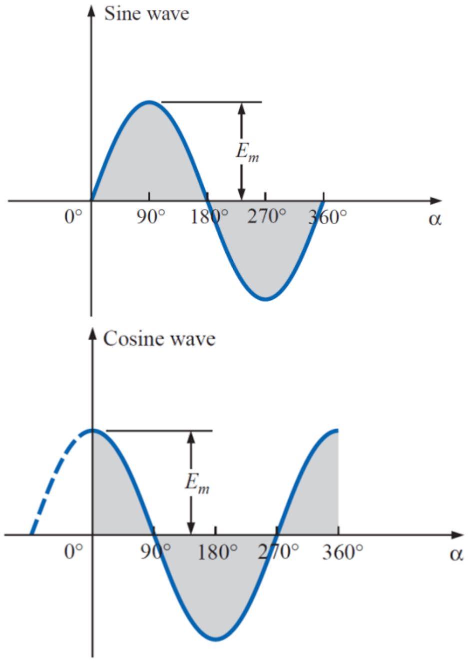
Solution:

From the figure, T = (25 ms - 5 ms) = 20 ms, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = 50\ Hz$$

THE SINE WAVE

The unit of measurement for the horizontal axis of Figure below is the *degree*. A second unit of measurement frequently used is the **radian** (rad).



$2\pi\ (rad) = 360\ (degree)$

$(rad) = \frac{degree}{360} 2\pi$

$(degree) = \frac{rad}{2\pi} 360$

$\pi = 3.14159$

$30^0 = \frac{\pi}{6}\ rad$

$45^0 = \frac{\pi}{4}\ rad$

$60^0 = \frac{\pi}{3}\ rad$

$90^0 = \frac{\pi}{2}\ rad$

The velocity with which the radius vector rotates about the centre, called the **angular velocity,** can be determined from the following equation:

$$\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}}$$

$$\omega = \frac{\alpha}{t}$$

$$\alpha = \omega t$$

For sinusoidal waveform, the angular velocity can be expressed as

$$\omega = \frac{2\pi}{T} \qquad \text{(rad/s)}$$

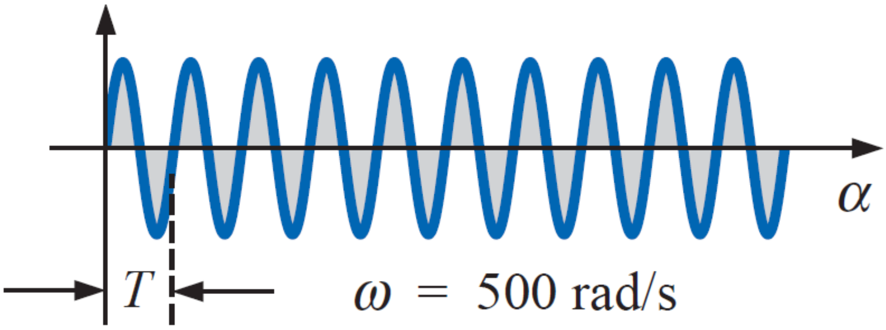
Example:
Determine the angular velocity of a sine wave having a frequency of 60 Hz.

Solution
$\omega = 2\pi f = 2 \times 3.14 \times 60 = 377\ rad/s$

Example:

Determine the frequency and period of the sine wave of following Figure.

Solution



$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{500} = 12.57\ ms$

$f = \frac{1}{T} = 79.58\ Hz$

Example:

Given $\omega = 200$ rad/s, determine how long it will take the sinusoidal waveform to pass through an angle of 90°.

$t = \frac{\alpha}{\omega} = \frac{\pi/2}{500} = 7.85\ ms$

Example:

Find the angle through which a sinusoidal waveform of 60 Hz will pass in a period of 5 ms.

$\alpha = \omega t = 2\pi f t = 2 \times 3.14 \times 60 \times 5 \times 10^{-3} = 1.885\ rad$

$\alpha(\ ^0) = \frac{180}{\pi} 1.885 = 108^0$

GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is

$i = I_m \sin\alpha = I_m \sin\omega t$

$v = V_m \sin\alpha = V_m \sin\omega t$

Example

Given $E = 5\sin\alpha$, determine E at $\alpha = 40^0$ and $\alpha = 0.8\pi$.

Solution:

$\alpha = 40^0$

$E = 5\sin\alpha$= 5(0.6428) = **3.214 V**

For $\alpha = 0.8\pi$,

α (°) =$\frac{180}{\pi}$ (0.8π) = 144°

and $E = 5\sin 144°$ = 5(0.5878) = **2.939 V**

The angle at which a particular voltage level is attained can be determined by rearranging the equation

$v = V_m sin\alpha$

$sin\alpha = \frac{v}{V_m}$

$\alpha = sin^{-1}\frac{v}{V_m}$

Similarly, for a particular current level,

$$\alpha = \sin^{-1}\frac{i}{I_m}$$

Example

a. Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.

b. Determine the time at which the magnitude is attained.

Solutions:

$$\alpha_1 = \sin^{-1}\frac{v}{E_m} = \sin^{-1} 0.4 = \mathbf{23.578°}$$

The magnitude of 4 V (positive) will be attained at two points between 0° and 180°. The second intersection is determined by

$$\alpha_2 = \mathbf{180° - 23.578° = 156.422°}$$

For the first intersection,

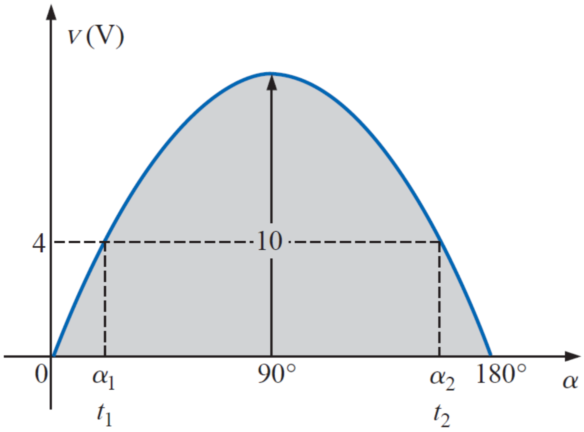
$$\alpha\ (rad) = \frac{\pi}{180} \times \mathbf{23.578} = 0.411$$

$$t_1 = \frac{\alpha}{\omega} = \frac{0.411}{377} = \mathbf{1.09\ ms}$$

For the second intersection,

$$\alpha\ (rad) = \frac{\pi}{180} \times \mathbf{156.422} = 2.73$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73}{377} = \mathbf{7.24\ ms}$$

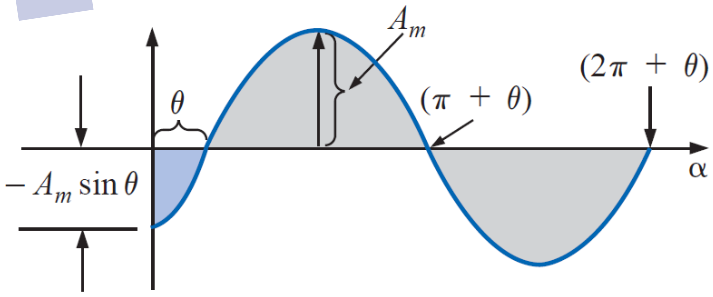


PHASE RELATIONS

If the waveform is shifted to the right or left of 0°, the expression becomes

$$A_m \sin(\omega t \pm \theta)$$

where θ is the angle in degrees or radians that the waveform has been shifted.



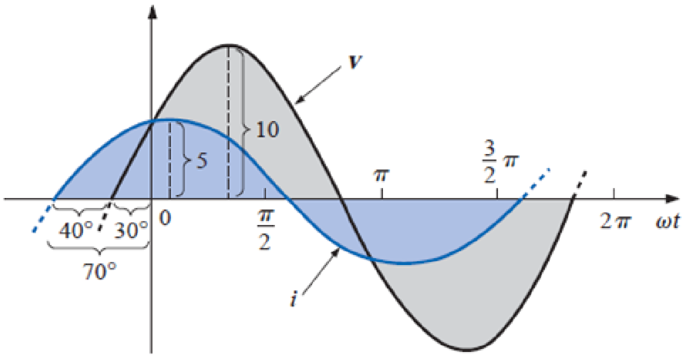
Example:

What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10\sin(wt + 30)$

$i = 5\sin(wt + 70)$

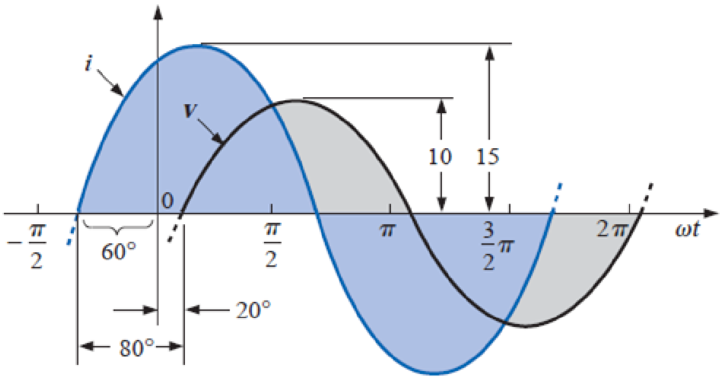
***i* leads *v* by 40°, or *v* lags *i* by 40°.**



b. $i = 15\sin(wt + 60)$

$v = 10\sin(wt - 20)$

i leads v by 80°, or v lags i by 80°.



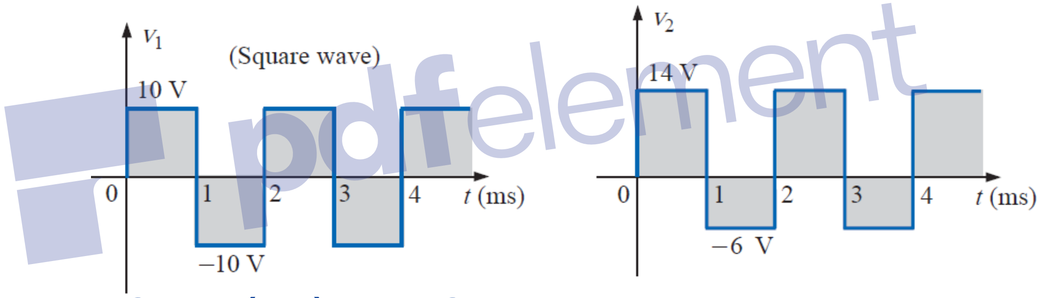
AVERAGE VALUE

For half wave rectifier

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} v(t)\, dt$$

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t\, d\omega t = \frac{2V_m}{\pi} = 0.636V_m$$

Example:
Determine the average value of the waveforms



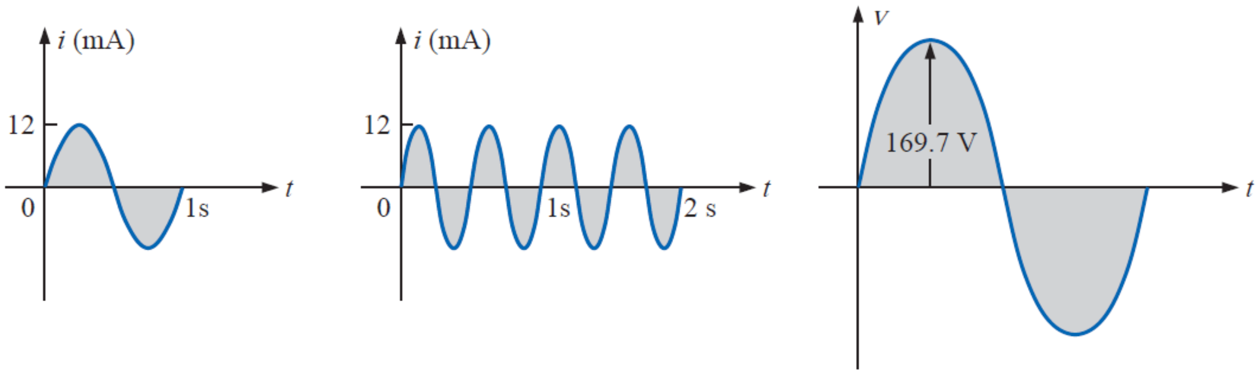
EFFECTIVE Root Mean Square (rms) VALUES

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(t)\, dt}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} {V_m}^2 \sin^2\omega t\, d\omega t} = \frac{V_m}{\sqrt{2}} = 0.707V_m$$

Example:

Find the rms values of the sinusoidal waveform in each part



RESPONSE OF BASIC *R, L,* AND *C* ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

Resistor

Let $v(t) = V_m \sin \omega t$

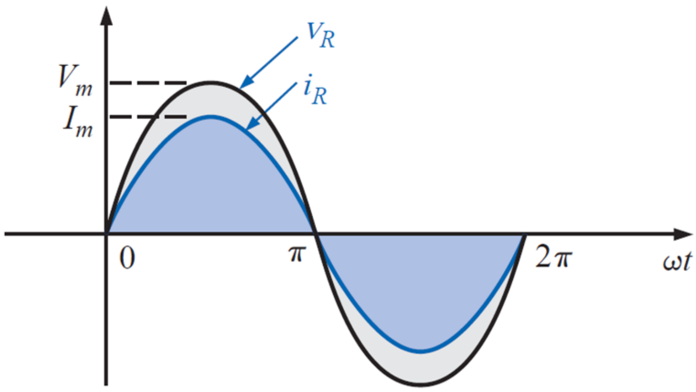
$$i(t) = \frac{v(t)}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

$$I_m = \frac{V_m}{R}$$

In addition, for a given $i(t) = I_m \sin \omega t$,

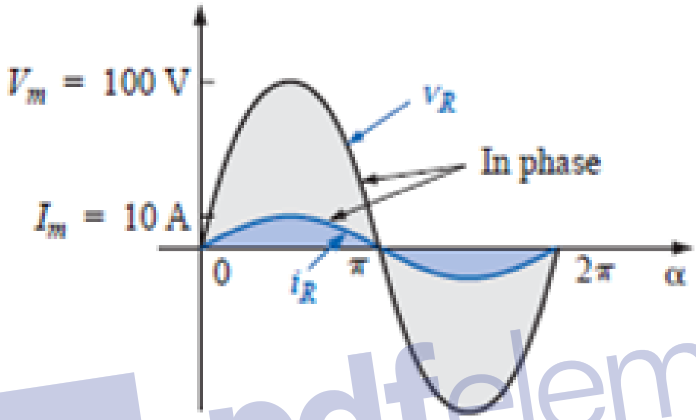
$v(t) = RI_m \sin \omega t = V_m \sin \omega t$

$V_m = RI_m$



Example:

The voltage across 10 Ω resistor is 100 $sin377t$, sketch the curves for the voltage and current.



Example:

The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ω. Sketch the curves for *v* and *i*.

a. *v* =100 sin 377*t*

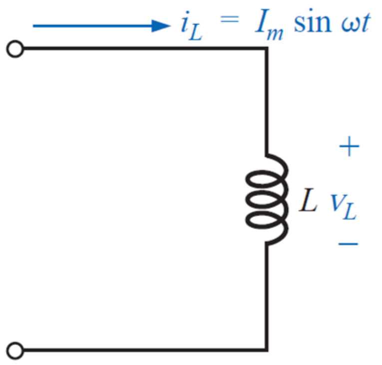
b. *v* = 25 sin(377*t* + 60°)

Inductor

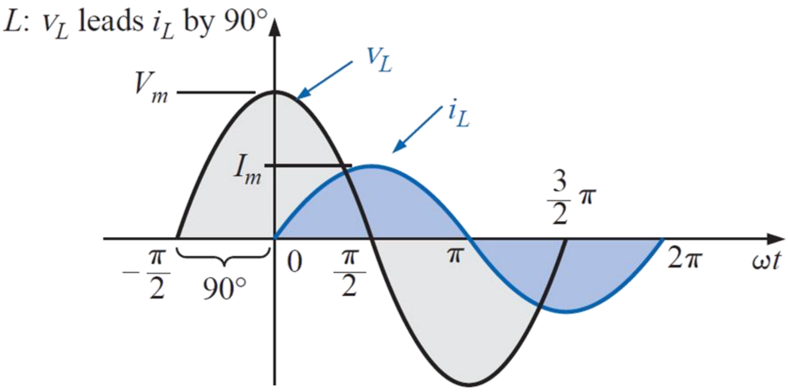
$$v_L = L\frac{di_L}{dt} = L\omega I_m \cos \omega t$$

$$v_L = V_m \cos \omega t = V_m \sin(\omega t + 90)$$

$$V_m = L\omega I_m$$



for an inductor, v_L leads i_L by 90°, or i_L lags v_L by 90°.



Reactance X_L

$$X_L = \frac{V_m}{I_m} = \frac{\omega L I_m}{I_m} = \omega L$$

Example:
The voltage across a 0.5-H coil is provided below. What is the sinusoidal expression for the current?
$v = 100 \sin 20t$

$$X_L = \omega L = 20 \times 0.5 = 10\ \Omega$$

$$i = \frac{V_m}{X_L}\sin(20t - 90) = 10\sin(20t - 90)A$$

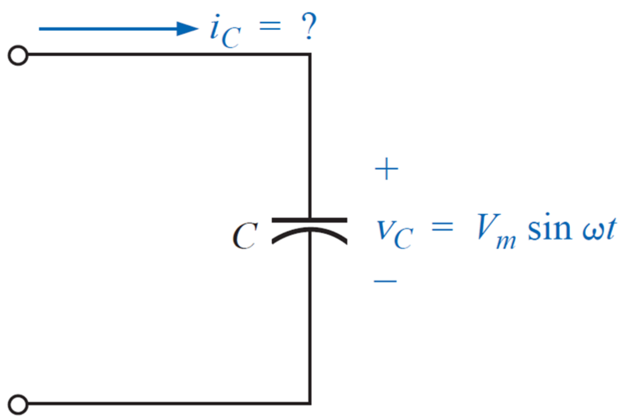
Example:
The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the *v* and *i* curves.
a. $i = 10 \sin 377t$
b. $i = 7 \sin(377t - 70°)$

Capacitor

$$i_C = C\frac{dv_C}{dt} = C\omega V_m \cos \omega t$$

$$i_C = I_m \cos \omega t = I_m \sin(\omega t + 90)$$

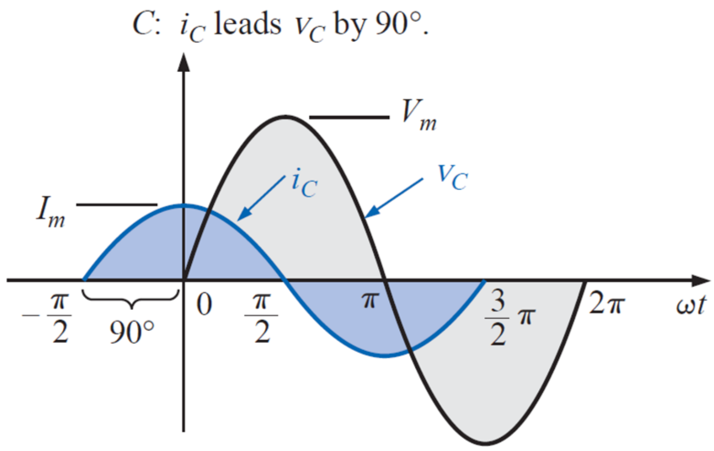
$$I_m = \omega C V_m$$



Reactance X_C

$$X_C = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

for a capacitor, i_C leads v_C by 90°, or v_C lags i_C by 90°.



Example:

The voltage across a 1-µF capacitor is provided below. What is the sinusoidal expression for the current? Sketch the *v* and *i* curves.

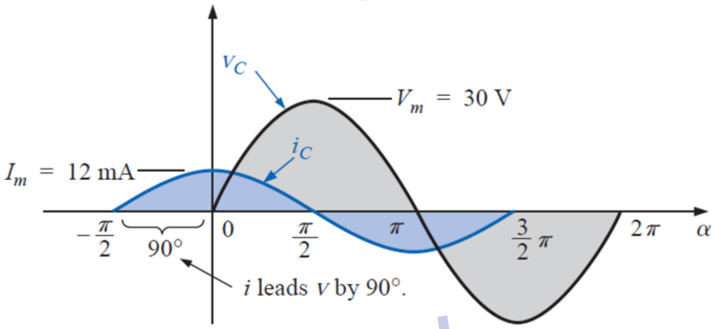
$\boldsymbol{v = 30 sin400t}$

Solution

$X_C = \frac{1}{\omega C} = \frac{1}{400 \times 1 \times 10^{-6}} = 2500\Omega$

$I_m = \frac{V_m}{\omega C} = \frac{30}{2500} = 12mA$

$v = 12sin(400t + 90)$



Example:

The current through a 100-mF capacitor is *i* = 40 sin(500*t* - 60°). Find the sinusoidal expression for the voltage across the capacitor.

Example:

For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor, and determine the value of *C*, *L*, or *R* if sufficient data are provided :

a. *v* = 100 sin(wt - 40°)
   i = 20 sin(wt - 40°)
b. *v* = 1000 sin(377*t* - 10°)
   i = 5 sin(377*t* - 80°)
c. *v* = 500 sin(157*t* - 30°)
   i = 1 sin(157*t* - 120°)
d. *v* = 50 cos(wt - 20°)
   i = 5 sin(wt - 110°)

AVERAGE POWER and power factor

Let we have

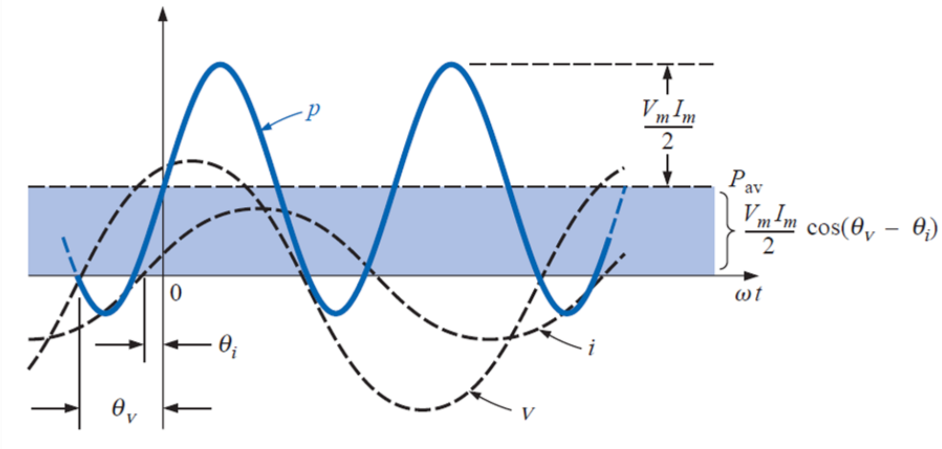
$v = V_m \sin(\omega t + \theta_v)$

$i = I_m \sin(\omega t + \theta_i)$

then the power is defined by

$p = vi = V_m \sin(\omega t + \theta_v)\, I_m \sin(\omega t + \theta_i)$

$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) - \frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i)$



$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{V_m I_m}{2}\cos\theta$$

$$\text{power factor} = F_p = \cos\theta = \frac{p}{\frac{V_m I_m}{2}} = \frac{p}{\frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}} = \frac{p}{V_{eff} I_{eff}}$$

Resistor

$$p = \frac{V_m I_m}{2}\cos(0) = \frac{V_m I_m}{2} = \frac{V_m I_m}{\sqrt{2}\sqrt{2}} = V_{eff} I_{eff} = \frac{V_{eff} V_{eff}}{R} = \frac{{V_{eff}}^2}{R} = {I_{eff}}^2 R$$

Inductor

$$p = \frac{V_m I_m}{2}\cos(90) = 0$$

Capacitor

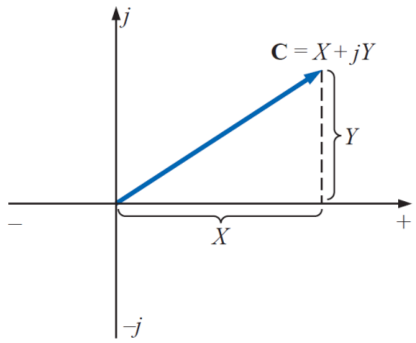
$$p = \frac{V_m I_m}{2}\cos(90) = 0$$

Example:
Find the average power dissipated in a network whose input current and voltage are the following:
$i = 5\sin(wt + 40°)$
$v = 10\sin(wt + 40°)$

Example:
Determine the average power delivered to networks having the following input voltage and current:
a. $v = 100\sin(wt + 40°)$
$i = 20\sin(wt + 70°)$
b. $v = 150\sin(wt - 70°)$
$i = 3\sin(wt - 50°)$

Example:
Determine the power factors of the following loads, and indicate whether they are leading or lagging:
a. $v = 50\sin(wt - 20°)$
$i = 2\sin(wt + 40°)$
b. $v = 120\sin(wt + 80°)$
$i = 5\sin(wt + 30°)$
c. $I_{eff} = 5A, V_{eff} = 20\ V\ and\ p = 100\ w$

COMPLEX NUMBERS



RECTANGULAR FORM

The format for the **rectangular form** is

$C = X + jY$

$X = Z\ cos\theta$

$y = Z\ sin\theta$

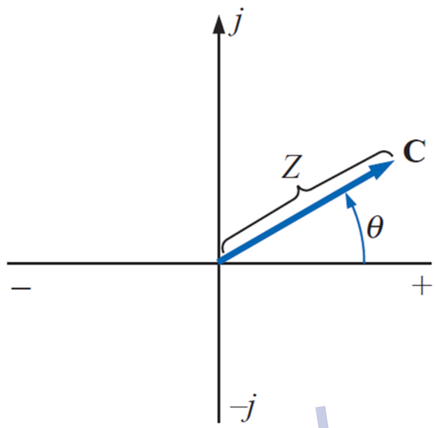
Example:

Sketch the following complex numbers in the complex plane:

a. **C** = 3 + j 4

b. **C** = 0 - j 6

c. **C** = -10 - j20



POLAR FORM

$C = Z\angle\theta$

$Z = \sqrt{X^2 + Y^2}$ $\quad \theta = tan^{-1}\dfrac{Y}{X}$

Example:

Sketch the following complex numbers in the complex plane:

a. **C** = $5\angle 30°$

b. **C** =$7\angle -120°$

c. **C** =$-4.2\angle 60°$

Example:

Convert the following from rectangular to polar form:

a. **C** = 3 + j 4

b. **C** = -6 + j 3

Example:

Convert the following from polar to rectangular form:

a. **C** = $10\angle 40°$

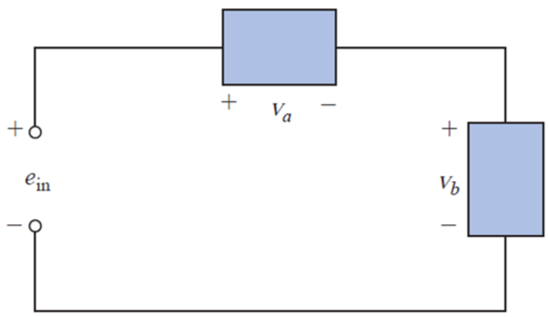
b. **C** = $10\angle 230°$

Example:

Find the input voltage of the circuit shown below when the frequency is 60 Hz

$v_a = 50\sin(377t + 30)$

$v_b = 30\sin(377t + 60)$



Solution:

$$v_a = \frac{50}{\sqrt{2}}\angle 30 = 35.35\angle 30\ V = 30.61\ \text{V} + j\ 17.68\ \text{V}$$
$$v_b = \frac{30}{\sqrt{2}}\angle 60 = 21.21\angle 60\ V = 10.61\ \text{V} + j\ 18.37\ \text{V}$$

Applying Kirchhoff's voltage law, we have

Rectangular form

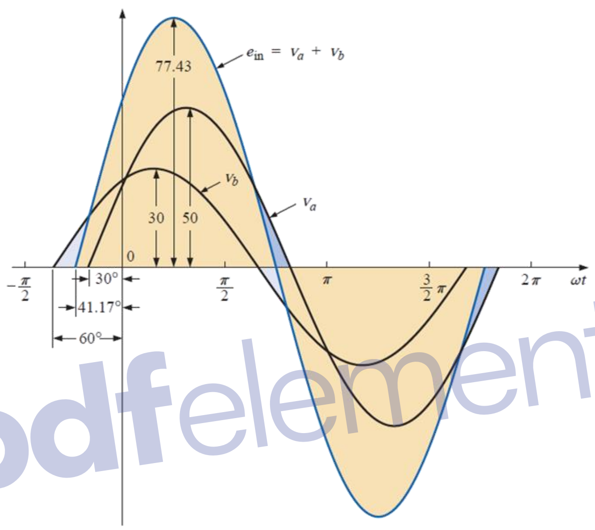
$$E_{in} = v_a + v_b = 30.61\ \text{V} + j\ 17.68 + 10.61\ \text{V} + j\ 18.37 = 41.22\ \text{V} + j\ 36.05\ \text{V}$$

Polar form

$$E_{in} = 54.76\ \text{V}\angle 41.17°\ \text{V}$$

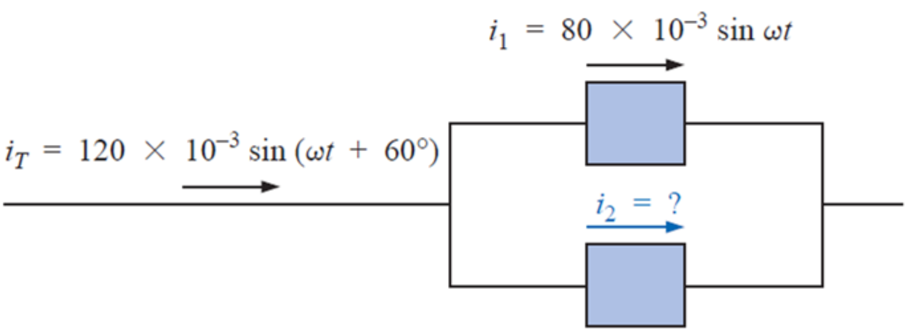
Time domain

$$E_{in} = \sqrt{2}(54.76)\sin(377t + 41.17) = 77.43\ \sin(377t + 41.17)$$



Example:

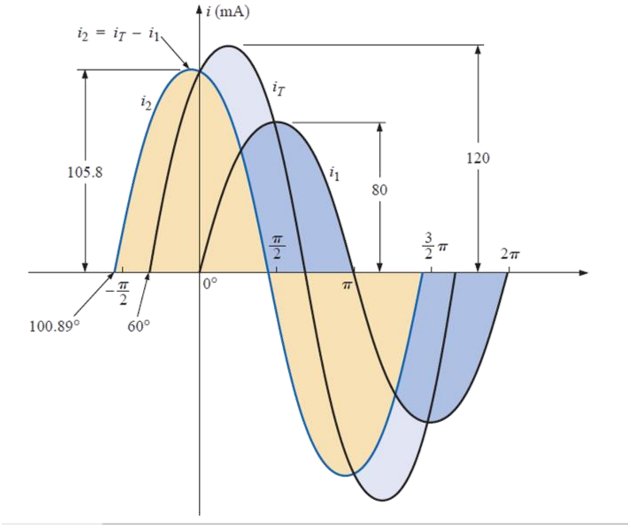
Determine the current i_2 for the network



Solution:

Applying Kirchhoff's current law, we obtain

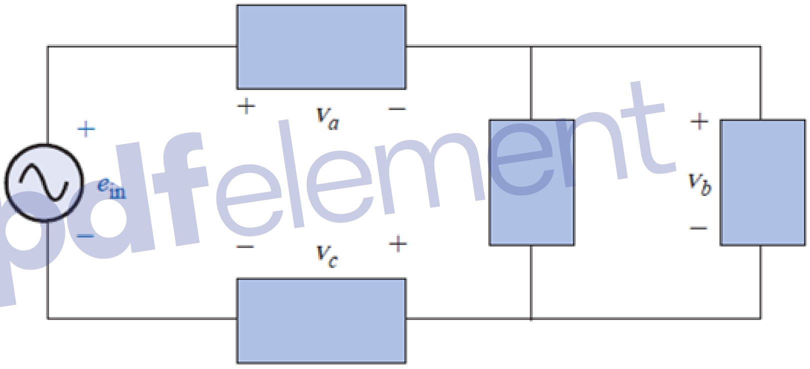
$$i_T = i_1 + i_2$$
$$i_2 = i_T - i_1$$
$$i_T = 120 \times 10^{-3}\sin(\omega t + 60) = \frac{120 \times 10^{-3}}{\sqrt{2}}\angle 60 = 84.84\angle 60\ mA = 42.42 + j\ 73.47\ \text{mA}$$
$$i_1 = 80 \times 10^{-3}\sin \omega t = \frac{80 \times 10^{-3}}{\sqrt{2}}\angle 0 = 56.56\angle 0\ mA = 56.56 + j\ 0\ \text{mA}$$
$$i_2 = i_T - i_1 = -14.14 + j\ 73.47\ \text{mA} = 74.82\ \text{mA}\angle 100.89° = 105.8 \times 10^{-3}\sin(\omega t + 100.89)\ A$$



H.W

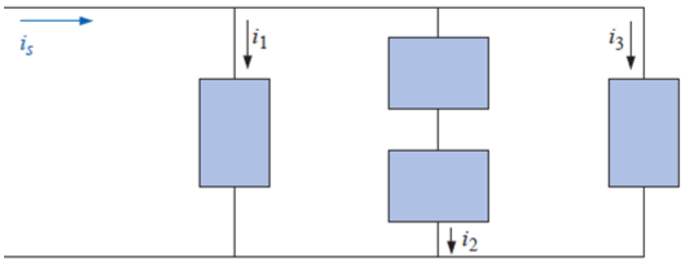
Find the sinusoidal expression for the applied voltage *e* for the system

$v_a = 60\sin(\omega t + 30)$
$v_b = 30\sin(\omega t - 30)$
$v_c = 40\sin(\omega t + 120)$



Find the sinusoidal expression for the current i_s for the system

$i_1 = 120 \times 10^{-3}\sin(377t + 180)$
$i_2 = 120 \times 10^{-3}\sin(377t)$
$i_3 = 2i_1$



MATHEMATICAL OPERATIONS WITH COMPLEX NUMBERS

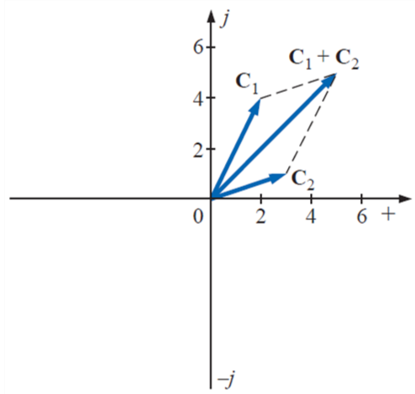
$j = \sqrt{-1}$
$j^2 = -1$
$\frac{1}{j} = -j$

Complex Conjugate

The **conjugate** or **complex conjugate** of a complex number can be found by simply changing the sign of the imaginary part in the rectangular form or by using the negative of the angle of the polar form. For example, the conjugate of $C = 2 + j3$ is $2 - j3$
The conjugate of **C** = $2\angle 30°$ is $2\angle -30°$

Addition

To add two or more complex numbers, simply add the real and imaginary parts separately. For example, if
$C_1 = X_1 + jY_1$ and $C_2 = X_2 + jY_2$
Then $C_1 + C_2 = X_1 + X_2 + j(Y_1 + Y_2)$

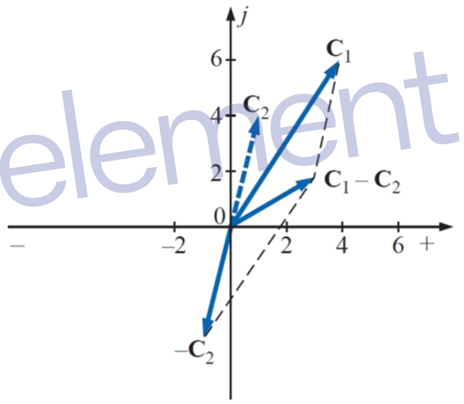


Example:

Add $C_1 = 2 + j4$ and $C_2 = 3 + j1$
Add $C_1 = 3 + j6$ and $C_2 = -6 + j3$

Subtraction

In subtraction, the real and imaginary parts are again considered separately. For example, if
$C_1 = X_1 + jY_1$ and $C_2 = X_2 + jY_2$
Then $C_1 - C_2 = X_1 - X_2 + j(Y_1 - Y_2)$



Example:

Subtract $C_2 = 1 + j4$ and $C_1 = 4 + j6$
Subtract $C_2 = -2 + j5$ and $C_1 = 3 + j3$

Multiplication

To multiply two complex numbers in *rectangular* form, multiply the real and imaginary parts of one in turn by the real and imaginary parts of the other. For example, if
$C_1 = X_1 + jY_1$ and $C_2 = X_2 + jY_2$
Then $C_1 \times C_2 = X_1X_2 - Y_1Y_2 + j(X_1Y_2 + X_2Y_1)$
Example:
Find **C**1 . **C**2 if
C1 = 2 + *j* 3 and **C**2 = 5 + *j* 10

In *polar* form, the magnitudes are multiplied and the angles added algebraically. For example, for
$C_1 = Z_1\angle\theta_1$ and $C_2 = Z_2\angle\theta_2$
Then $C_1.C_2 = Z_1Z_2\angle\theta_1 + \theta_2$
Example:
Find **C**1 . **C**2 if $C_1 = 5\angle 20$ and $C_2 = 10\angle 30$

Division

To divide two complex numbers in *rectangular* form, multiply the numerator and denominator by the conjugate of the denominator and the resulting real and imaginary parts collected. That is, if

$C_1 = X_1 + jY_1$ and $C_2 = X_2 + jY_2$

Then

$$\frac{C_1}{C_2} = \frac{X_1 + jY_1}{X_2 + jY_2} = \frac{X_1 + jY_1}{X_2 + jY_2} \times \frac{X_2 - jY_2}{X_2 - jY_2} = \frac{X_1X_2 + Y_1Y_2}{X_2^2 + Y_2^2} + j\frac{X_2Y - X_1Y_2}{X_2^2 + Y_2^2}$$

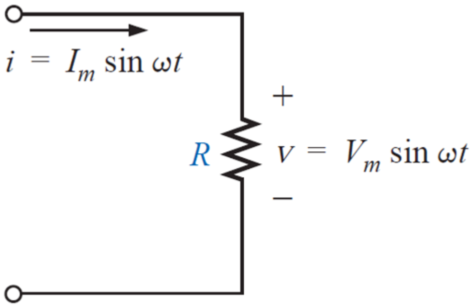
Example:

Find **C**1 / **C**2 if

C1 = 1 + *j* 4 and **C**2 = 4 + *j* 5

Series and Parallel ac Circuits

Resistive Elements



$$I_m = \frac{V_m}{R}$$
$$V_m = I_m R$$

In phaser form,

$$v = V_m sin\omega t = V\angle 0$$

Where $V = 0.707V_m$

Applying Ohm's law and using phaser algebra, we have

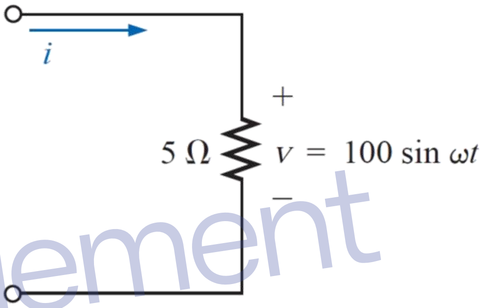
$$I = \frac{V\angle 0}{R\angle 0}$$

So that in the time domain,

$$i = \sqrt{2}\frac{V}{R}sin\omega t$$

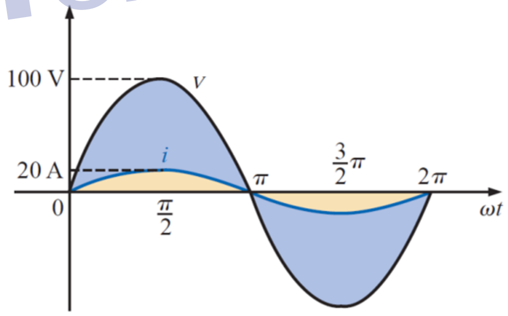
Example:

Using complex algebra, find the current *i* for the circuit shown below. Sketch the waveforms of *v* and *i*.



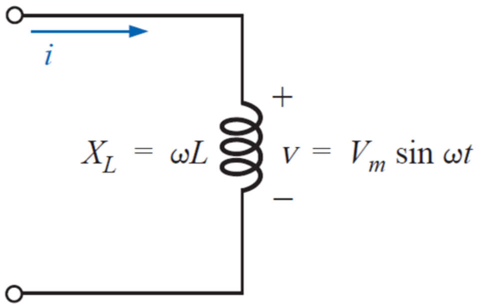
Solution

$$v = 100sin\omega t = 70.7\angle 0$$
$$I = \frac{V\angle 0}{Z_R\angle 0} = \frac{70.7\angle 0}{5\angle 0} = 14.14\angle 0\ A$$
$$i = \sqrt{2} \times 14.14\ sin\omega t = 20\ sin\omega t\ A$$



Inductive Reactance

The voltage leads the current by 90° and that the reactance of the coil *XL* is determined by ωL.



$$v = V_m sin\omega t = V\angle 0$$

By Ohm's law,

$$\mathbf{I} = \frac{V\angle 0}{X_L\angle 90} = \frac{V}{X_L}\angle -90$$

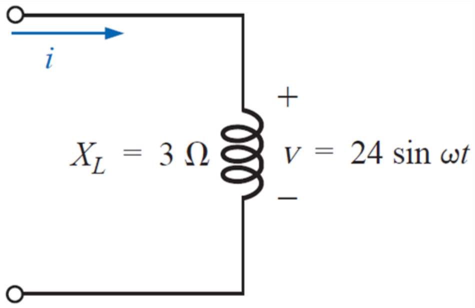
so that in the time domain,

$$i = \sqrt{2}\frac{V}{X_L}sin(\omega t - 90)$$

$Z_L = X_L\angle 90$

Example:

Using complex algebra, find the current *i* for the circuit shown below. Sketch the *v* and *i* curves.



Solution:

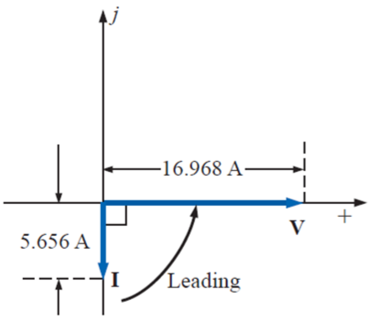
$v = 24 sin\omega t$

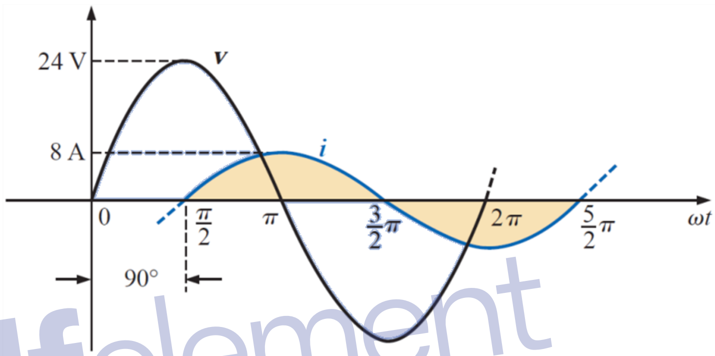
In polar form

$\mathbf{V} = 16.968\angle 0$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z_L}} = \frac{V\angle 0}{X_L\angle 90} = \frac{16.968\angle 0}{3\angle 90} = 5.656\ \text{A}\angle -90$$

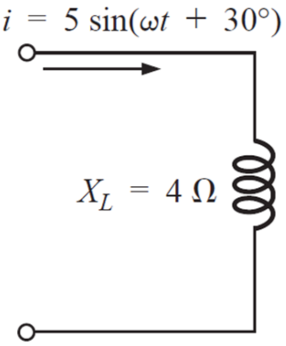
$$i = \sqrt{2}(5.656)\ sin(\omega t - 90) = 8\ sin(\omega t - 90)$$





Example:

Using complex algebra, find the voltage *v* for the circuit shown below. Sketch the *v* and *i* curves.



Capacitive Reactance

The current leads the voltage by 90° and that the reactance of the capacitor X_C is determined by $\frac{1}{\omega C}$.

$v = V_m sin\omega t$

In polar form

$\mathbf{V} = V\angle 0$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z_C}} = \frac{V\angle 0}{X_C\angle -90} = \frac{V}{X_C}\angle 90$$

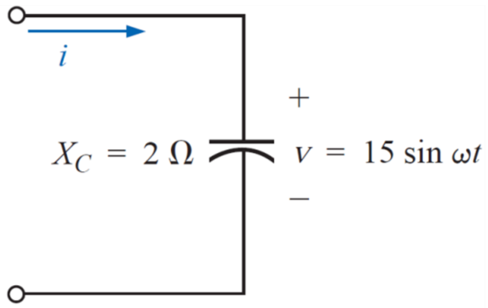
$$i = \sqrt{2}\frac{V}{X_C} sin(\omega t + 90)$$

$\mathbf{Z_C} = X_C\angle -90$

Example:

Using complex algebra, find the current *i* for the circuit shown below. Sketch the *v* and *i* curves.

solution:



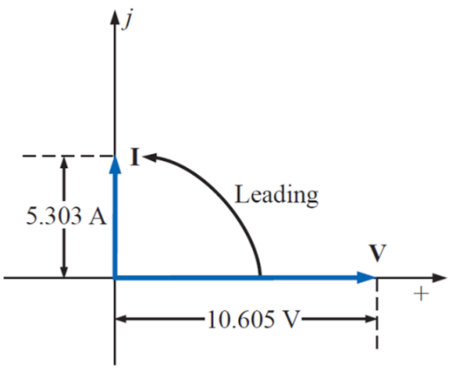
$v = 15\ sin\omega t$

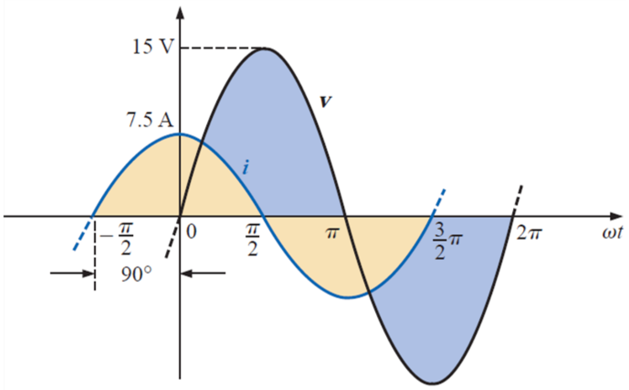
In polar form

$\mathbf{V} = 10.605\ \angle 0$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z_C}} = \frac{V\angle 0}{X_C\angle -90} = \frac{10.605\ \angle 0}{2\angle -90} = 5.303\ \mathrm{A}\angle 90$$

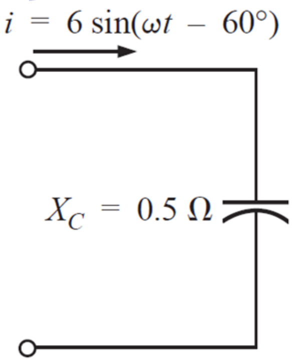
$$i = \sqrt{2}\frac{V}{X_C}\sin(\omega t + 90) = 7.5\ sin(\omega t + 90)$$





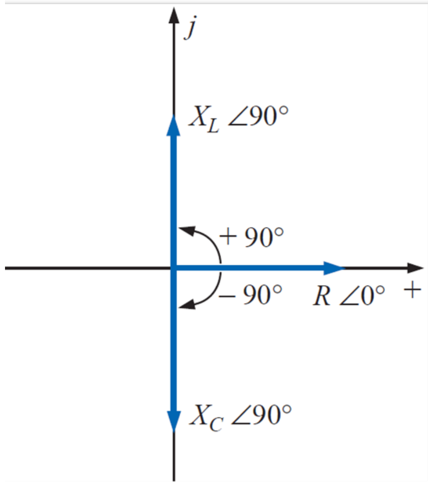
Example:

Using complex algebra, find the voltage v for the circuit shown below. Sketch the *v* and *i* curves.



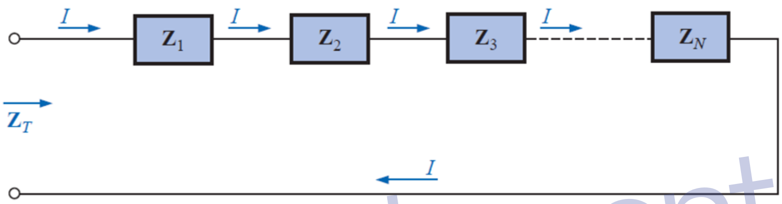
Impedance Diagram

Now that an angle is associated with resistance, inductive reactance, and capacitive reactance, each can be placed on a complex plane diagram. For any network, the resistance will *always* appear on the positive real axis, the inductive reactance on the positive imaginary axis, and the capacitive reactance on the negative imaginary axis. The result is an **impedance diagram** that can reflect the individual and total impedance levels of an ac network.



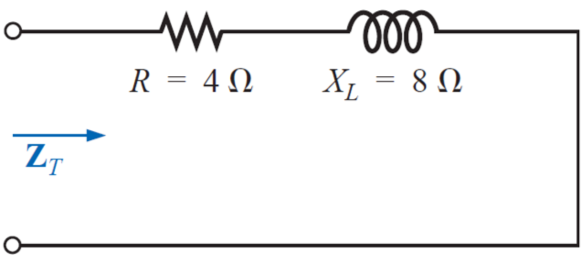
SERIES CONFIGURATION

The overall properties of series ac circuits are the same as those for dc circuits. For instance, the total impedance of a system is the sum of the individual impedances:

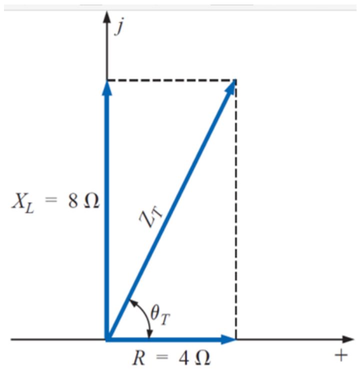


$$Z_T = Z_1 + Z_2 + \cdots .. + Z_N$$

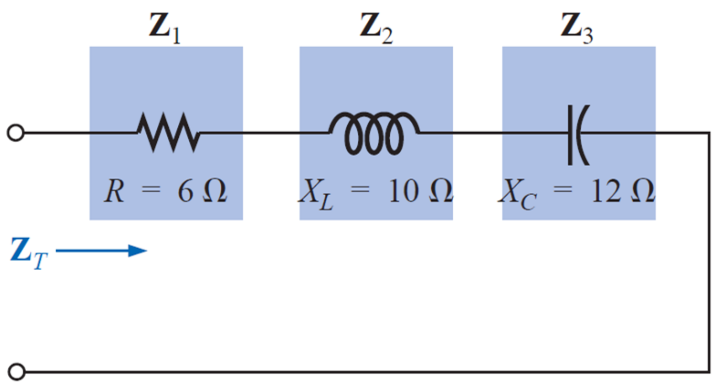
Example:
Draw the impedance diagram for the circuit shown below, and find the total impedance.



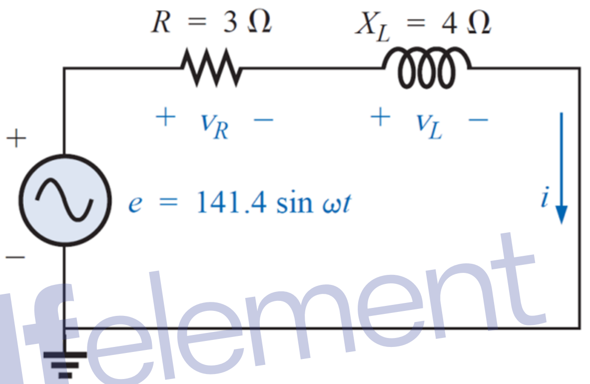
$$\mathbf{Z_T} = \mathbf{Z_1} + \mathbf{Z_2} = R + jX_L = 4 + j8 = \mathbf{8.944\angle 63.43°\ \Omega}$$



Example:
Determine the input impedance to the series network shown below. Draw the impedance diagram.



R-L



Phasor Notation

$e = 141.4\,sin\omega t$

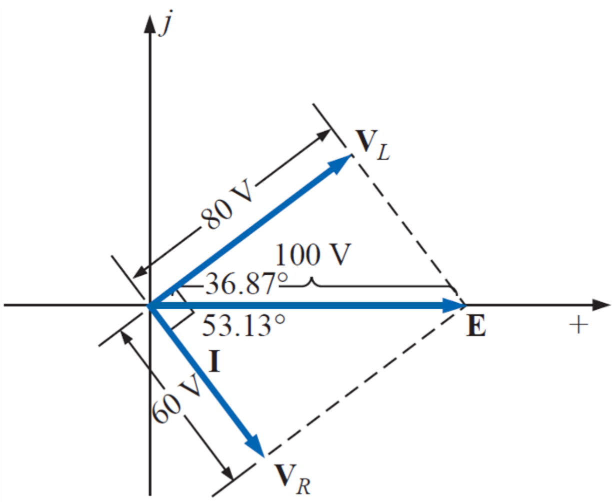
$\mathbf{E} = 100\angle 0$

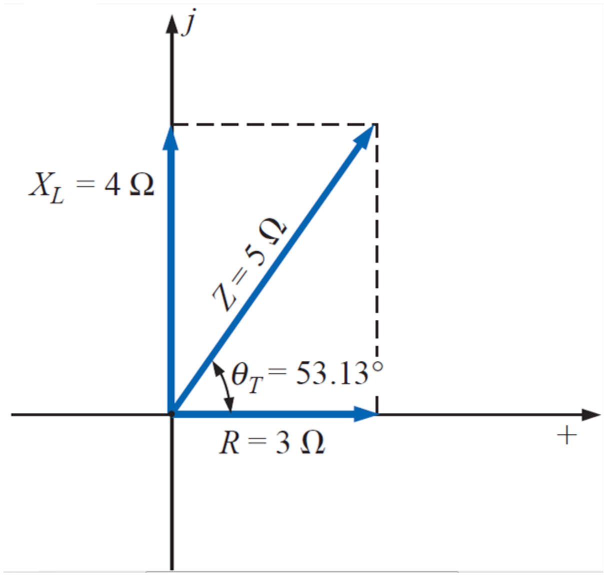
$\mathbf{Z_T} = \mathbf{Z_1} + \mathbf{Z_2} = R + jX_L = 3 + j4 = \mathbf{5\angle 53.13°\ \Omega}$

$\mathbf{I} = \dfrac{\mathbf{E}}{\mathbf{Z_T}} = \dfrac{100\angle 0}{\mathbf{5\angle 53.13°}} = \mathbf{20\angle -53.13°\ A}$

$\mathbf{V}_R = \mathbf{IZ_R} = \mathbf{3 \times 20\angle -53.13° = 60\angle -53.13°} = 36 - j\,48\ \mathbf{V}$

$\mathbf{V}_L = \mathbf{IZ_L} = \mathbf{4\angle 90 \times 20\angle -53.13° = 80\angle 36.87°} = 64 + j\,48\ \mathbf{V}$





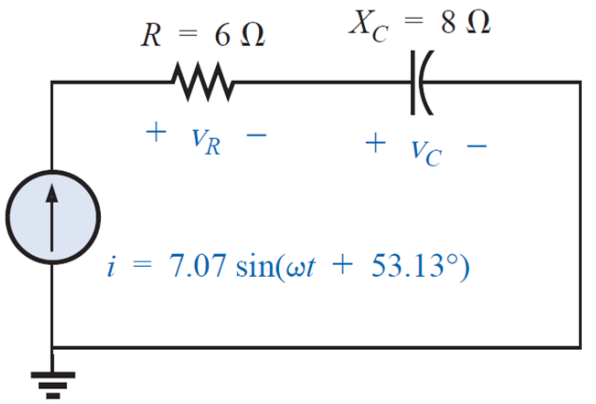
Power: The total power in watts delivered to the circuit is
$p_T = EIcos\theta_T = 100 \times 20 \cos 53.13° = 1200$ w
where E and I are effective values and θ_T is the phase angle between E and I, or
$p_T = I^2R = 20^2 \times 3 = 1200$ w
where I is the effective value, or, finally,
$p_T = p_R + p_L = 60 \times 20\ cos0 + 80 \times 20 \cos 90 = 1200$ w

Power factor: The power factor Fp of the circuit is cos 53.13° =**0.6 lagging,** where 53.13° is the phase angle between **E** and **I.**

$$cos\theta = \frac{p}{EI} = \frac{I^2R}{EI} = \frac{IR}{E} = \frac{R}{E/I} = \frac{R}{Z_T}$$

R-C

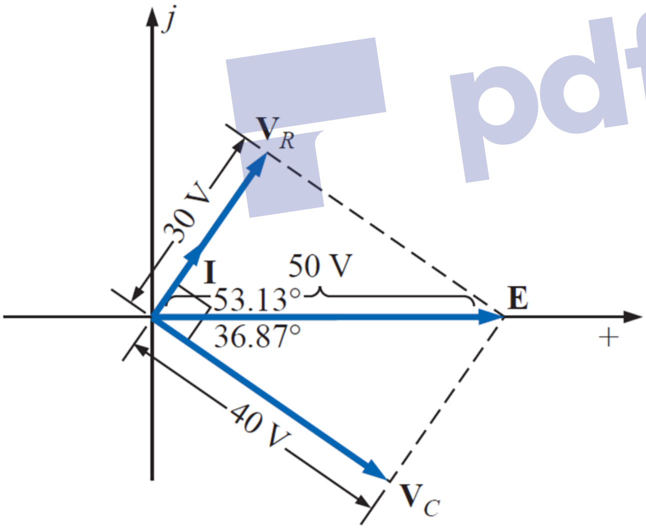
Phasor Notation

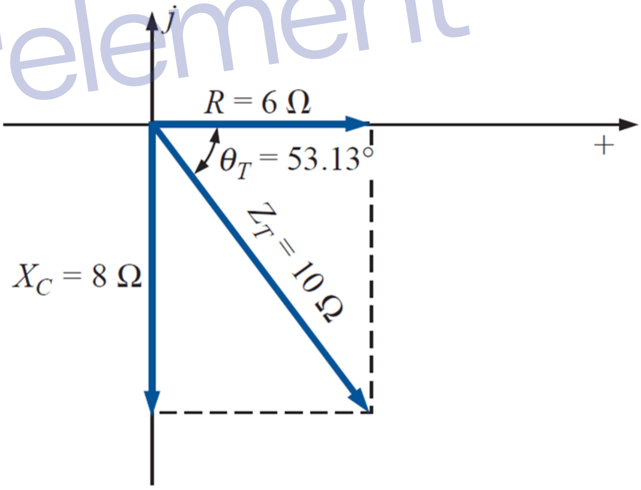


$i = 7.07\ sin(\omega t + 53.13°)$
$\mathbf{I} = \mathbf{5\angle 53.13°\ A}$

$\mathbf{Z_T} = \mathbf{Z_1} + \mathbf{Z_2} = R - jX_C = 6 - j8 = \mathbf{10\angle - 53.13°\ \Omega}$
$\mathbf{E} = \mathbf{IZ_T} = \mathbf{5\angle 53.13°} \times \mathbf{10\angle - 53.13°} = \mathbf{50\angle 0°}$

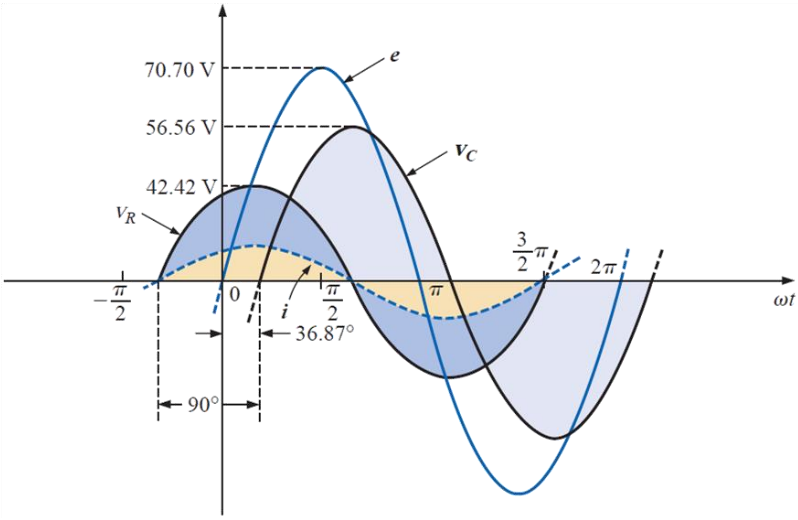
$\mathbf{V_R} = \mathbf{IZ_R} = \mathbf{5\angle 53.13°} \times \mathbf{6\angle 0°} = \mathbf{30\angle 53.13\ V}$
$\mathbf{V_L} = \mathbf{IZ_L} = \mathbf{5\angle 53.13°} \times \mathbf{8\angle - 90°} = \mathbf{40\angle - 36.87°\ V}$





Time domain: In the time domain,

$e = \sqrt{2} \times 50 \sin \omega t = 70.7\ \sin \omega t$
$V_R = \sqrt{2} \times 30 \sin(\omega t + \mathbf{53.13°}) = 42.42\ \sin(\omega t + \mathbf{53.13°})$
$V_C = \sqrt{2} \times 40 \sin(\omega t - 36.87) = 56.56 \sin(\omega t - 36.87)$



Power: The total power in watts delivered to the circuit is

$p_T = EIcos\theta_T = 50 \times 5\cos 53.13° = 150$ w

where E and I are effective values and θ_T is the phase angle between E and I, or

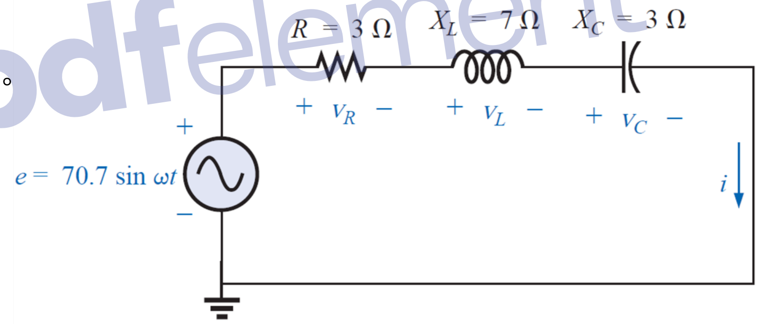
$p_T = I^2R = 5^2 \times 6 = 150$ w

$p_T = p_R + p_C = 30 \times 5\ cos0 + 40 \times 5\cos 90 = 150$ w

Power factor: The power factor of the circuit is

$F_P = \cos 53.13° = \mathbf{0.6\ leading}$

R L C



$\mathbf{Z_T} = \mathbf{Z_R} + \mathbf{Z_L} + \mathbf{Z_C} = R + jX_L - jX_C$

$\mathbf{Z_T} = 3 + j7 - j3 = 3 + j4 = \mathbf{5\angle 53.13°}$

Impedance diagram

$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z_T}} = \frac{50\angle 0}{\mathbf{5\angle 53.13°}} = \mathbf{10\angle -53.13°\ A}$

$\mathbf{V}_R = \mathbf{IZ_R} = \mathbf{3 \times 10\angle -53.13° = 30\angle -53.13°\ V}$

$\mathbf{V}_L = \mathbf{IZ_L} = \mathbf{7\angle 90 \times 10\angle -53.13° = 70\angle 36.87°\ V}$

$\mathbf{V}_C = \mathbf{IZ_C} = \mathbf{3\angle -90 \times 10\angle -53.13° = 30\angle -143.13°\ V}$

Phasor diagram: The phasor diagram of Fig. 15.38 indicates that the current **I** is in phase with the voltage across the resistor, lags the voltage across the inductor by 90°, and leads the voltage across the capacitor by 90°.

Time domain:

$i = \sqrt{2} \times 10\sin(\omega t - 53.13°) = 14.14\ \sin(\omega t - 53.13°)$

$V_R = \sqrt{2} \times 30\sin(\omega t - 53.13°) = 42.42\ \sin(\omega t - 53.13°)$

$V_L = \sqrt{2} \times 70\sin(\omega t + 36.87) = 98.98\sin(\omega t + 36.87)$

$V_C = \sqrt{2} \times 30\sin(\omega t - 143.13°) = 42.42\sin(\omega t - 143.13°)$

Power: The total power in watts delivered to the circuit is

$p_T = EIcos\theta_T = 50 \times 10\cos 53.13° = 300$ w

or

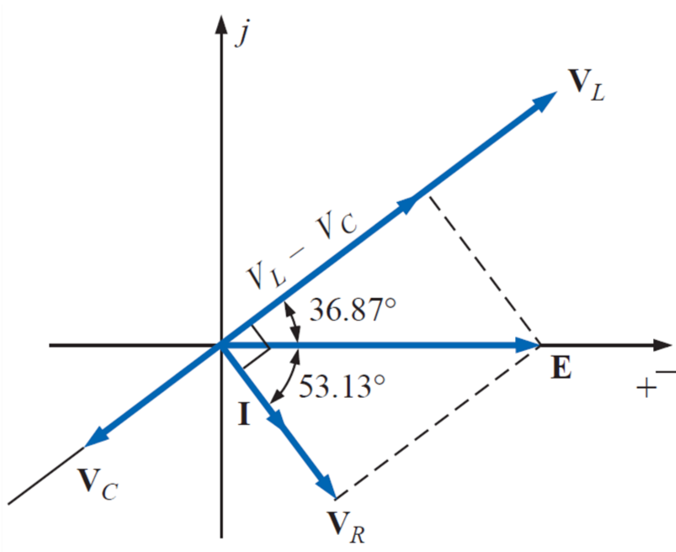
$p_T = I^2R = 10^2 \times 3 = 300$ w

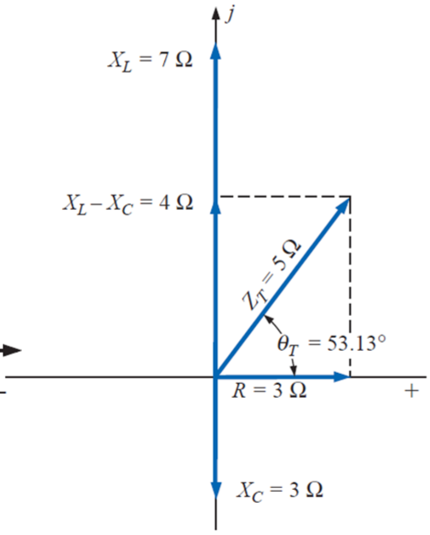
$p_T = p_R + p_L + p_C = 30 \times 10\ cos0 + 70 \times 10 \cos 90 + 40 \times 10 \cos 90 = 300$ w

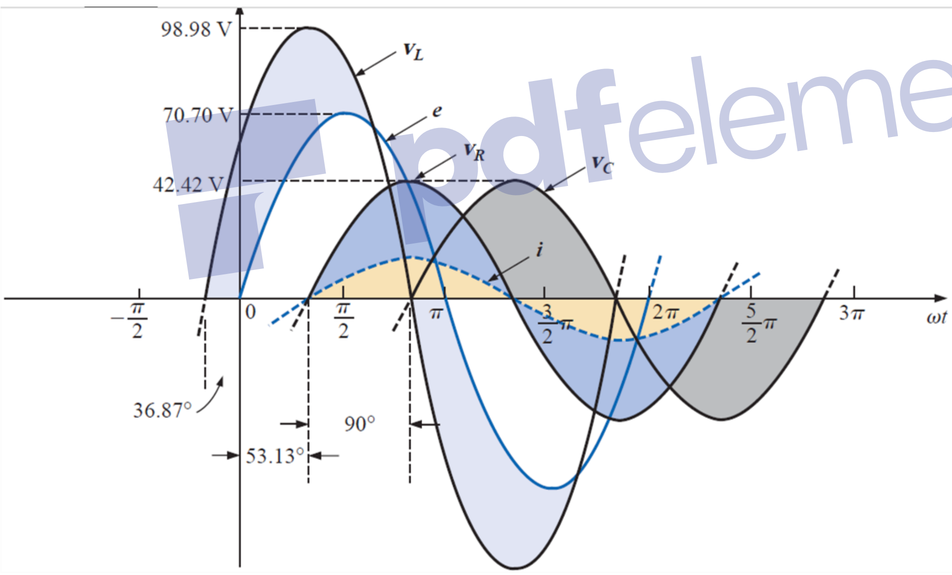
Power factor: The power factor of the circuit is

$F_P = \cos 53.13° = \mathbf{0.6\ leading}$

$$F_P = \frac{R}{\mathbf{Z_T}} = \frac{3}{\mathbf{5}} = \mathbf{0.6\ leading}$$



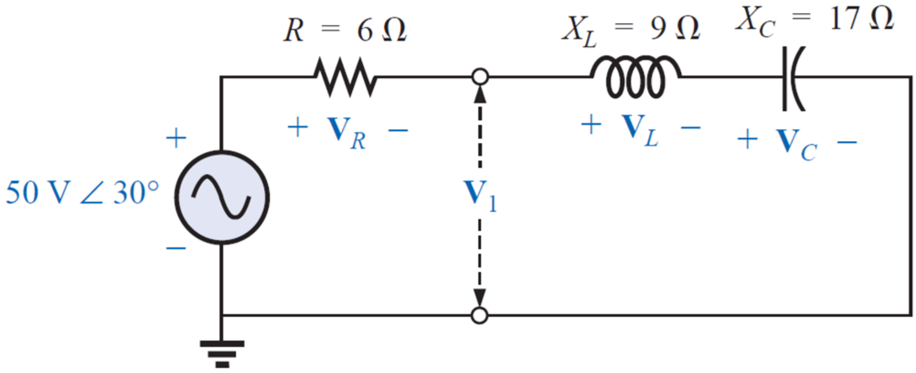




VOLTAGE DIVIDER RULE

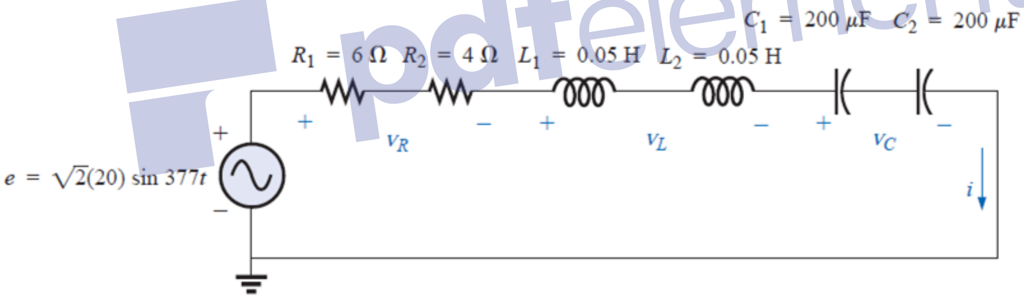
The basic format for the **voltage divider rule** in ac circuits is exactly the same as that for dc circuits:

Example:
Find the voltage across each element of the circuit shown below



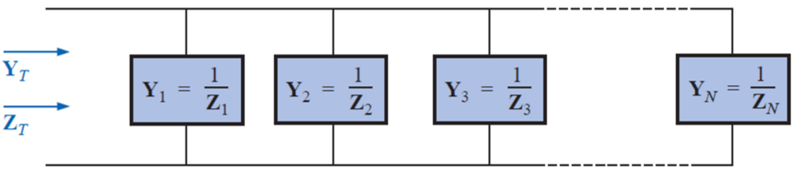
H.W
For the circuit shown below,

1- Calculate **I**, **V**$_R$, **V**$_L$, and **V**$_C$ in phasor form.
2- Calculate the total power factor.
3- Calculate the average power delivered to the circuit.
4- Draw the phasor diagram.
5- Obtain the phasor sum of V_R, V_L, and V_C, and show that it equals the input voltage E.
6- Find V_R and V_C using the voltage divider rule.



PARALLEL ac CIRCUITS

In ac circuits, we define **admittance** (**Y**) as being equal to 1/**Z**. The unit of measure for admittance as defined by the SI system is *siemens*, which has the symbol S. Admittance is a measure of how well an ac circuit will *admit*, or allow, current to flow in the circuit. The larger its value, therefore, the heavier the current flow for the same applied potential. The total admittance of a circuit can also be found by finding the sum of the parallel admittances.



$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N}$$
$$Y_T = Y_1 + Y_2 + \cdots + Y_N$$

For pure resistor, conductance is the reciprocal of resistance, and

$$Y_R = \frac{1}{Z_R} = \frac{1}{R\angle 0} = G\angle 0 \quad \text{(siemens, S)}$$

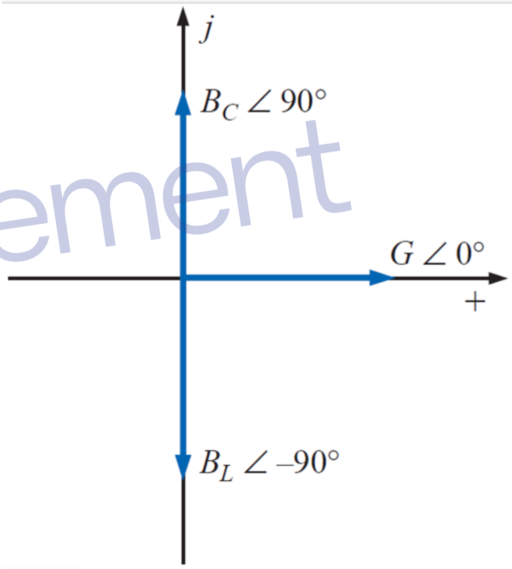
The reciprocal of reactance (1/X) is called **susceptance** and is a measure of how *susceptible* an element is to the passage of current through it. Susceptance is also measured in *siemens* and is represented by the capital letter B.
For the inductor,

$$Y_L = \frac{1}{Z_L} = \frac{1}{X_L\angle 90} = B_L\angle -90 \quad \text{(siemens, S)}$$

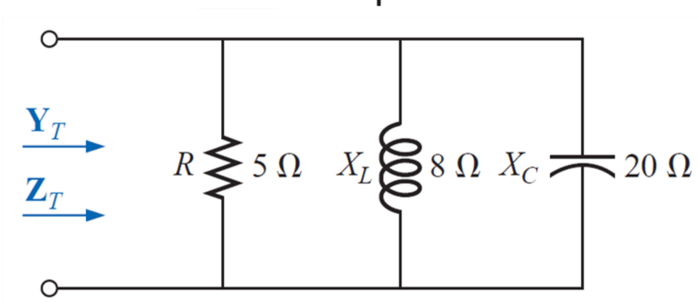
Note that for inductance, an increase in frequency or inductance will result in a decrease in susceptance or, correspondingly, in admittance.
For the capacitor,

$$Y_C = \frac{1}{Z_C} = \frac{1}{X_C\angle -90} = B_C\angle 90 \quad \text{(siemens, S)}$$

For the capacitor, therefore, an increase in frequency or capacitance will result in an increase in its susceptibility.
For parallel ac circuits, the **admittance diagram** is used with the three admittances, represented as shown in Figure below.
Note from this figure that the conductance (like resistance) is on the positive real axis, whereas inductive and capacitive susceptances are in direct opposition on the imaginary axis.



Example:
For the network of Fig. shown below:
a. Find the admittance of each parallel branch.
b. Determine the input admittance.
c. Calculate the input impedance.
d. Draw the admittance diagram.



Solution:

$$\text{a. } \mathbf{Y}_R = G\angle 0° = \frac{1}{R}\angle 0° = \frac{1}{5\,\Omega}\angle 0°$$

$$= \mathbf{0.2\ S\ \angle 0° = 0.2\ S + \mathit{j}\ 0}$$

$$\mathbf{Y}_L = B_L \angle -90^\circ = \frac{1}{X_L} \angle -90^\circ = \frac{1}{8\ \Omega} \angle -90^\circ$$

$$= \mathbf{0.125\ S \angle -90^\circ = 0 - j\,0.125\ S}$$

$$\mathbf{Y}_C = B_C \angle 90^\circ = \frac{1}{X_C} \angle 90^\circ = \frac{1}{20\ \Omega} \angle 90^\circ$$

$$= \mathbf{0.050\ S \angle +90^\circ = 0 + j\,0.050\ S}$$

b. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C$

$$= (0.2\ S + j\,0) + (0 - j\,0.125\ S) + (0 + j\,0.050\ S)$$

$$= 0.2\ S - j\,0.075\ S = \mathbf{0.2136\ S \angle -20.56^\circ}$$

c. $\mathbf{Z}_T = \dfrac{1}{0.2136\ S \angle -20.56^\circ} = \mathbf{4.68\ \Omega \angle 20.56^\circ}$

or

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L\mathbf{Z}_C}{\mathbf{Z}_R\mathbf{Z}_L + \mathbf{Z}_L\mathbf{Z}_C + \mathbf{Z}_R\mathbf{Z}_C}$$

$$= \frac{(5\ \Omega \angle 0^\circ)(8\ \Omega \angle 90^\circ)(20\ \Omega \angle -90^\circ)}{(5\ \Omega \angle 0^\circ)(8\ \Omega \angle 90^\circ) + (8\ \Omega \angle 90^\circ)(20\ \Omega \angle -90^\circ) + (5\ \Omega \angle 0^\circ)(20\ \Omega \angle -90^\circ)}$$

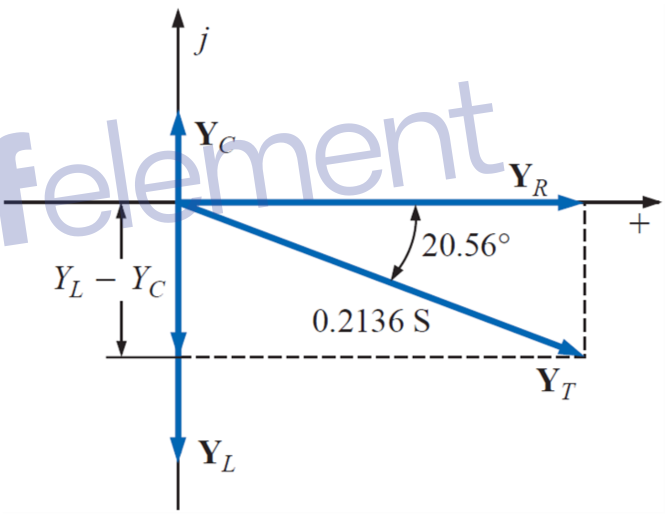
$$= \frac{800\ \Omega \angle 0^\circ}{40 \angle 90^\circ + 160 \angle 0^\circ + 100 \angle -90^\circ}$$

$$= \frac{800\ \Omega}{160 + j\,40 - j\,100} = \frac{800\ \Omega}{160 - j\,60}$$

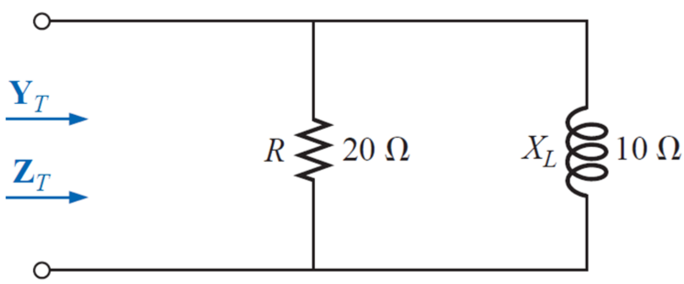
$$= \frac{800\ \Omega}{170.88 \angle -20.56^\circ}$$

$$= \mathbf{4.68\ \Omega \angle 20.56^\circ}$$

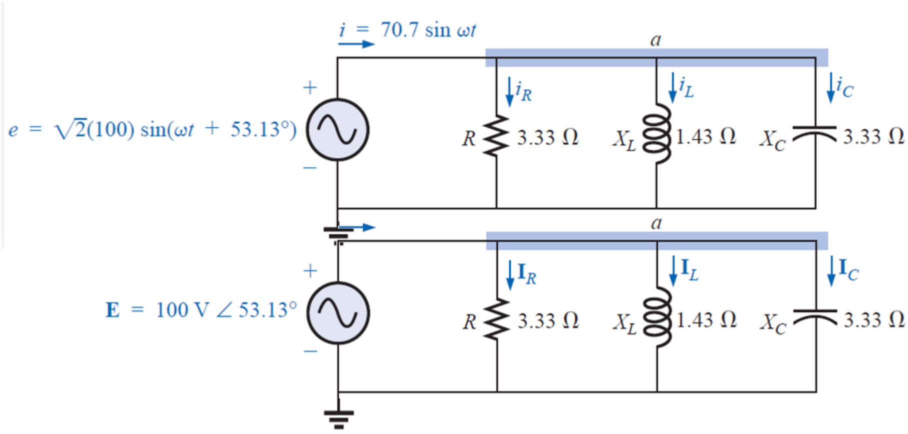
d. The admittance diagram



Example:
For the network of Fig. shown below:
a. Find the admittance of each parallel branch.
b. Determine the input admittance.
c. Calculate the input impedance.
d. Draw the admittance diagram.



PARALLEL ac NETWORKS



$\mathbf{Y}_T$ and $\mathbf{Z}_T$

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C = G\angle 0^\circ + B_L\angle -90^\circ + B_C\angle 90^\circ$$

$$= \frac{1}{3.33\ \Omega}\angle 0^\circ + \frac{1}{1.43\ \Omega}\angle -90^\circ + \frac{1}{3.33\ \Omega}\angle 90^\circ$$

$$= 0.3\ \text{S}\angle 0^\circ + 0.7\ \text{S}\angle -90^\circ + 0.3\ \text{S}\angle 90^\circ$$

$$= 0.3\ \text{S} - j\,0.7\ \text{S} + j\,0.3\ \text{S}$$

$$= 0.3\ \text{S} - j\,0.4\ \text{S} = \mathbf{0.5\ S\angle -53.13^\circ}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\ \text{S}\angle -53.13^\circ} = \mathbf{2\ \Omega\angle 53.13^\circ}$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (100\ \text{V}\angle 53.13^\circ)(0.5\ \text{S}\angle -53.13^\circ) = \mathbf{50\ A\angle 0^\circ}$$

$\mathbf{I}_R$, $\mathbf{I}_L$, and $\mathbf{I}_C$

$$\mathbf{I}_R = (E\angle\theta)(G\angle 0^\circ)$$
$$= (100\ \text{V}\angle 53.13^\circ)(0.3\ \text{S}\angle 0^\circ) = \mathbf{30\ A\angle 53.13^\circ}$$

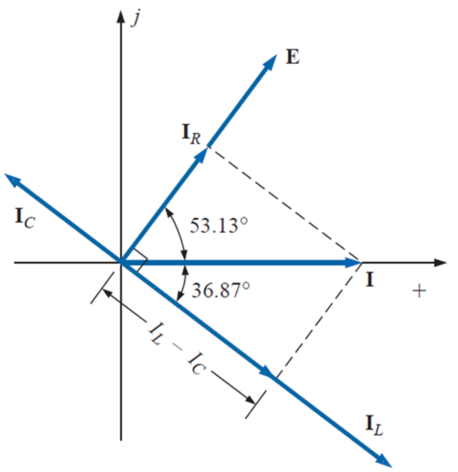
$$\mathbf{I}_L = (E\angle\theta)(B_L\angle -90^\circ)$$
$$= (100\ \text{V}\angle 53.13^\circ)(0.7\ \text{S}\angle -90^\circ) = \mathbf{70\ A\angle -36.87^\circ}$$

$$\mathbf{I}_C = (E\angle\theta)(B_C\angle 90^\circ)$$
$$= (100\ \text{V}\angle 53.13^\circ)(0.3\ \text{S}\angle +90^\circ) = \mathbf{30\ A\angle 143.13^\circ}$$

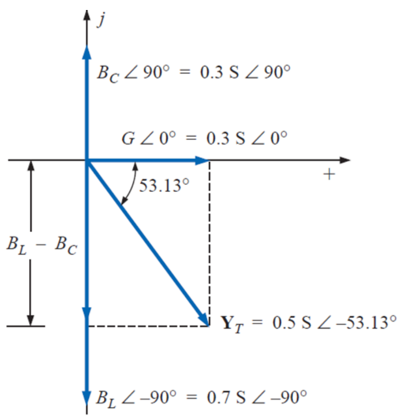
Kirchhoff's current law: At node *a*,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L - \mathbf{I}_C = 0$$

Phasor diagram



Admittance diagram:



Time domain:

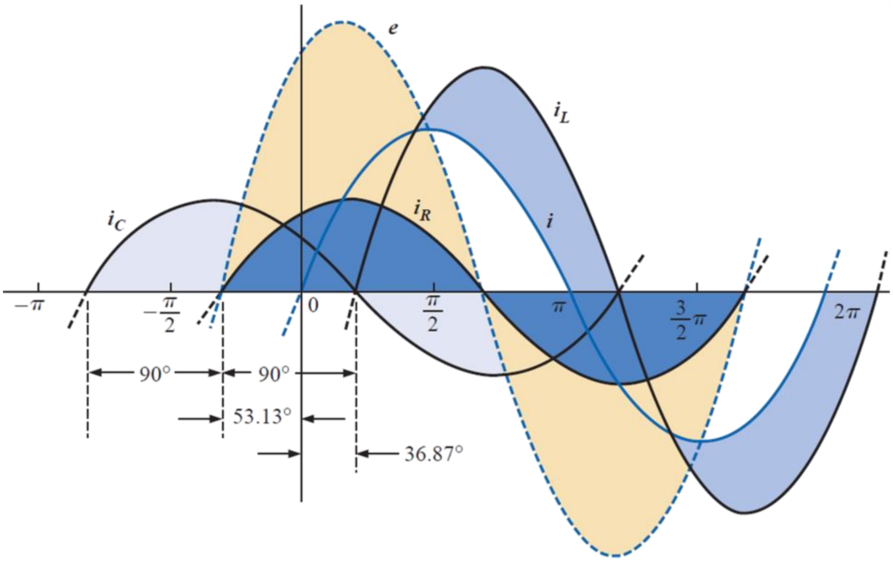
$$i = \sqrt{2}(50)\sin\omega t = \mathbf{70.70\ sin\ \omega t}$$

$$i_R = \sqrt{2}(30)\sin(\omega t + 53.13°) = \mathbf{42.42\ sin(\omega t + 53.13°)}$$

$$i_L = \sqrt{2}(70)\sin(\omega t - 36.87°) = \mathbf{98.98\ sin(\omega t - 36.87°)}$$

$$i_C = \sqrt{2}(30)\sin(\omega t + 143.13°) = \mathbf{42.42\ sin(\omega t + 143.13°)}$$

A plot of all the currents and the impressed voltages appears ain following figure



Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta = (100\ \text{V})(50\ \text{A})\cos 53.13° = (5000)(0.6)$$
$$= \mathbf{3000\ W}$$

or $$P_T = E^2G = (100\ \text{V})^2(0.3\ \text{S}) = \mathbf{3000\ W}$$

or, finally,

$$P_T = P_R + P_L + P_C$$
$$= EI_R\cos\theta_R + EI_L\cos\theta_L + EI_C\cos\theta_C$$
$$= (100\ \text{V})(30\ \text{A})\cos 0° + (100\ \text{V})(70\ \text{A})\cos 90° + (100\ \text{V})(30\ \text{A})\cos 90°$$
$$= 3000\ \text{W} + 0 + 0$$
$$= \mathbf{3000\ W}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

$$F_p = \cos\theta_T = \frac{G}{Y_T} = \frac{0.3\ \text{S}}{0.5\ \text{S}} = \mathbf{0.6\ lagging}$$

Impedance approach: The input current **I** can also be determined by first finding the total impedance in the following manner:

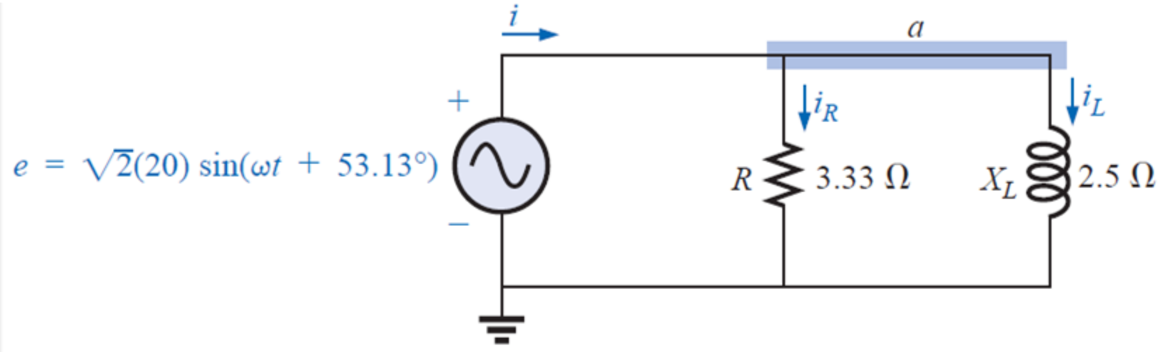
$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L\mathbf{Z}_C}{\mathbf{Z}_R\mathbf{Z}_L + \mathbf{Z}_L\mathbf{Z}_C + \mathbf{Z}_R\mathbf{Z}_C} = \mathbf{2\ \Omega\ \angle 53.13°}$$

and, applying Ohm's law, we obtain

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\ \text{V}\ \angle 53.13°}{2\ \Omega\ \angle 53.13°} = \mathbf{50\ A\ \angle 0°}$$

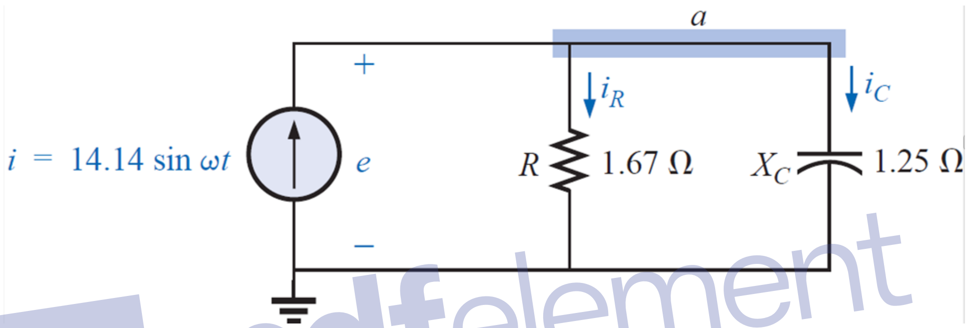
Example:

For the circuit shown below, determine the I_R and I_L , phasor and admittance diagrams, time domain representation, power and power factor.



Example:

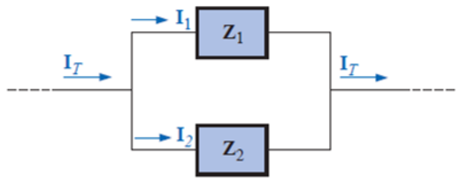
For the circuit shown below, determine the I_R and I_C , phasor and admittance diagrams, time domain representation, power and power factor.



CURRENT DIVIDER RULE

The basic format for the **current divider rule** in ac circuits is exactly the same as that for dc circuits; that is, for two parallel branches with impedances **Z**1 and **Z**2 as shown

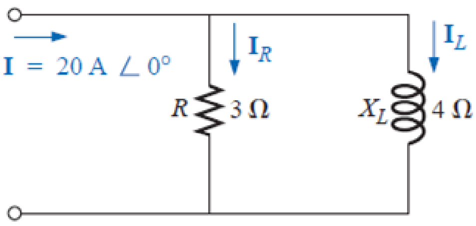
$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2}$$



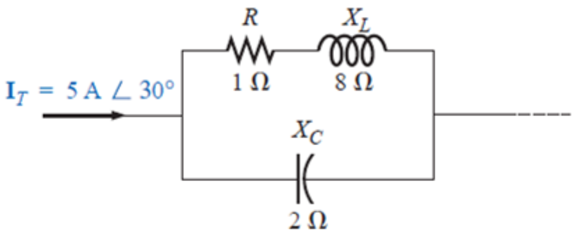
Example:

Using the current divider rule, find the current through each impedance of following Figure.

$$\mathbf{I}_R = \frac{\mathbf{Z}_L\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(4\ \Omega\ \angle 90°)(20\ \text{A}\ \angle 0°)}{3\ \Omega\ \angle 0° + 4\ \Omega\ \angle 90°} = \frac{80\ \text{A}\ \angle 90°}{5\ \angle 53.13°}$$
$$= \mathbf{16\ A\ \angle 36.87°}$$
$$\mathbf{I}_L = \frac{\mathbf{Z}_R\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(3\ \Omega\ \angle 0°)(20\ \text{A}\ \angle 0°)}{5\ \Omega\ \angle 53.13°} = \frac{60\ \text{A}\ \angle 0°}{5\ \angle 53.13°}$$
$$= \mathbf{12\ A\ \angle -53.13°}$$



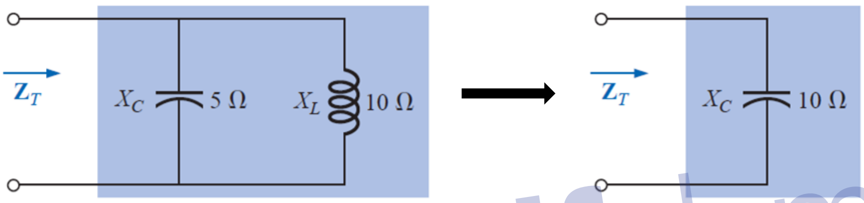
Example:
Using the current divider rule, find the current through each parallel branch of Figure shown below.

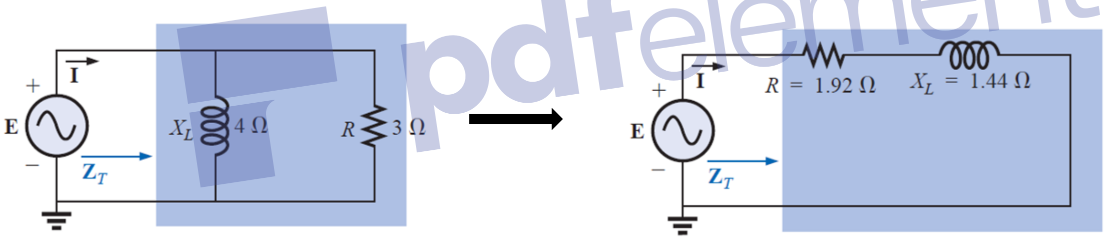


EQUIVALENT CIRCUITS

In a series ac circuit, the total impedance of two or more elements in series is often equivalent to an impedance that can be achieved with fewer elements of different values, the elements and their values being determined by the frequency applied. This is also true for parallel circuits.

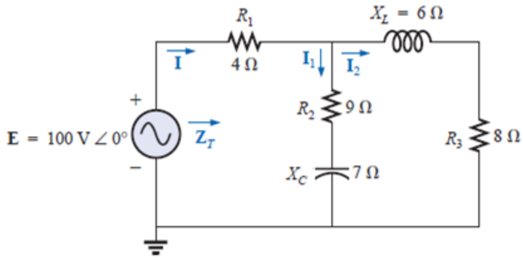
$$\mathbf{Z}_T = \frac{\mathbf{Z}_C\mathbf{Z}_L}{\mathbf{Z}_C + \mathbf{Z}_L} = \frac{(5\ \Omega\ \angle -90^\circ)(10\ \Omega\ \angle 90^\circ)}{5\ \Omega\ \angle -90^\circ + 10\ \Omega\ \angle 90^\circ} = \frac{50\ \angle 0^\circ}{5\ \angle 90^\circ}$$
$$= 10\ \Omega\ \angle -90^\circ$$





$$\mathbf{Z}_T = \frac{\mathbf{Z}_L\mathbf{Z}_R}{\mathbf{Z}_L + \mathbf{Z}_R} = \frac{(4\ \Omega\ \angle 90^\circ)(3\ \Omega\ \angle 0^\circ)}{4\ \Omega\ \angle 90^\circ + 3\ \Omega\ \angle 0^\circ}$$
$$= \frac{12\ \angle 90^\circ}{5\ \angle 53.13^\circ} = 2.40\ \Omega\ \angle 36.87^\circ$$
$$= 1.920\ \Omega + j\,1.440\ \Omega$$

Example:
For the following network



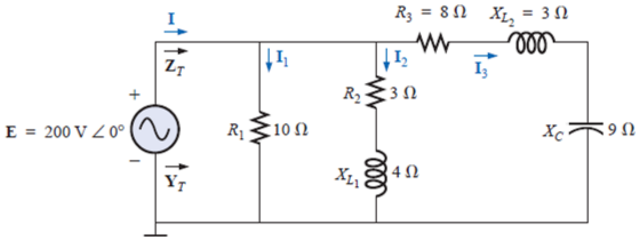
a. Calculate the total impedance $\mathbf{Z}_T$.
b. Compute **I**.
c. Find the total power factor.
d. Calculate $\mathbf{I}_1$ and $\mathbf{I}_2$.
e. Find the average power delivered to the circuit.

Example:
For the following network

a. Compute **I**.
b. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
c. Verify Kirchhoff's current law by showing that

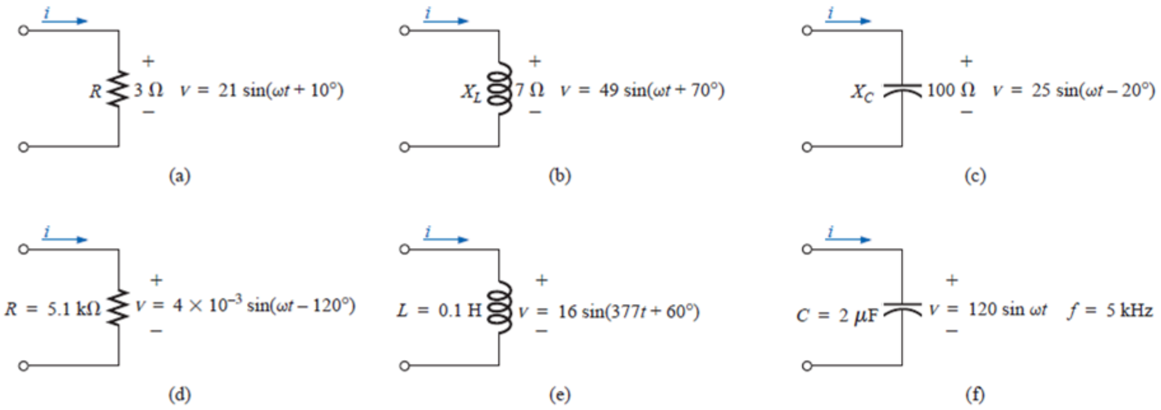
$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

d. Find the total impedance of the circuit.

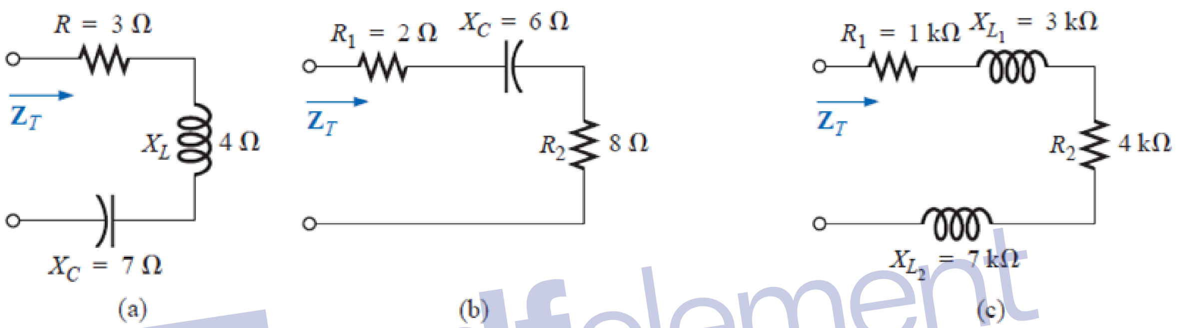


Tutorial:

1-Find the current i for the elements and sketch the waveforms for v and i on the same set of axes.

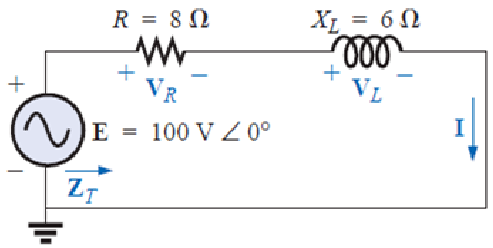


2-Calculate the total impedance and express your answer in rectangular and polar forms, and draw the impedance diagram.

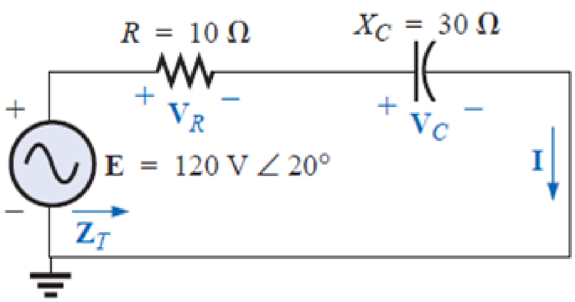


3-For the circuit shown below

a. Find the total impedance Z_T in polar form.
b. Draw the impedance diagram.
c. Find the current **I** and the voltages V_R and V_L in phasor form.
d. Draw the phasor diagram of the voltages **E**, V_R, and V_L, and the current **I**.
e. Verify Kirchhoff's voltage law around the closed loop.
f. Find the average power delivered to the circuit.
g. Find the power factor of the circuit, and indicate whether it is leading or lagging.
h. Find the sinusoidal expressions for the voltages and current if the frequency is 60 Hz.
i. Plot the waveforms for the voltages and current on the same set of axes.



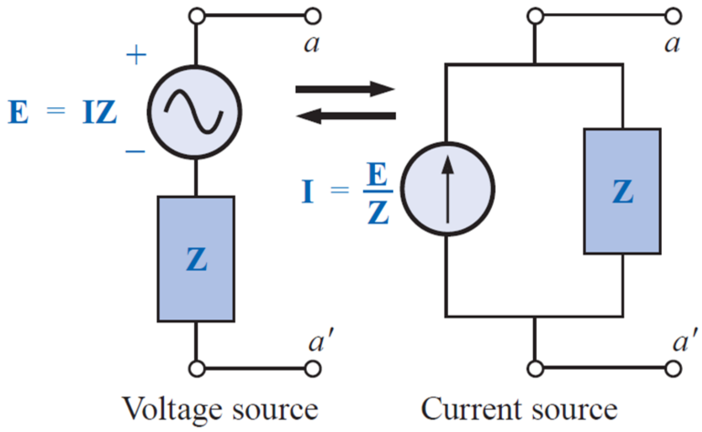
4-Repeat problem 3 for the following circuit, replacing V_L with V_C in parts (c) and (d).



Methods of Analysis

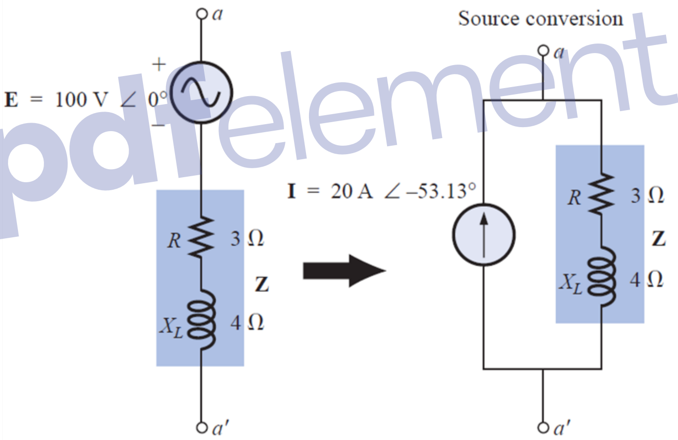
SOURCE CONVERSIONS

When applying the methods to be discussed, it may be necessary to convert a current source to a voltage source, or a voltage source to a current source. This **source conversion** can be accomplished in much the same manner as for dc circuits, except now we shall be dealing with phasors and impedances instead of just real numbers and resistors.



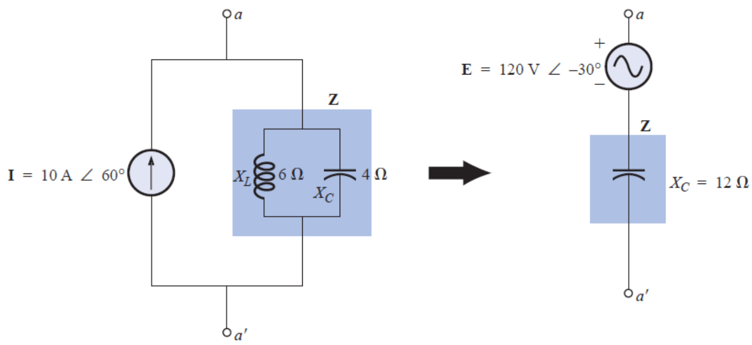
Example:
Convert the voltage source to a current source.

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{100\ \text{V}\ \angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ}$$
$$= \mathbf{20\ A\ \angle -53.13^\circ}$$



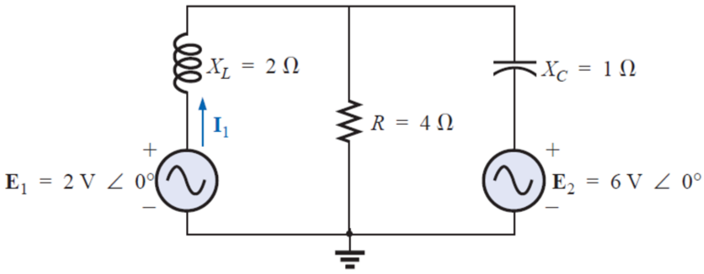
Example:
Convert the current source to a voltage source.

$$\mathbf{Z} = \frac{\mathbf{Z}_C \mathbf{Z}_L}{\mathbf{Z}_C + \mathbf{Z}_L} = \frac{(X_C \angle -90^\circ)(X_L \angle 90^\circ)}{-j\, X_C + j\, X_L}$$
$$= \frac{(4\ \Omega \angle -90^\circ)(6\ \Omega \angle 90^\circ)}{-j\, 4\ \Omega + j\, 6\ \Omega} = \frac{24\ \Omega \angle 0^\circ}{2 \angle 90^\circ}$$
$$= \mathbf{12\ \Omega \angle -90^\circ} \qquad \text{[Fig. 17.7(b)]}$$
$$\mathbf{E} = \mathbf{IZ} = (10\ \text{A} \angle 60^\circ)(12\ \Omega \angle -90^\circ)$$
$$= \mathbf{120\ V \angle -30^\circ}$$



MESH ANALYSIS

Example:
Using mesh analysis, find the current $\mathbf{I}_1$



Solution:

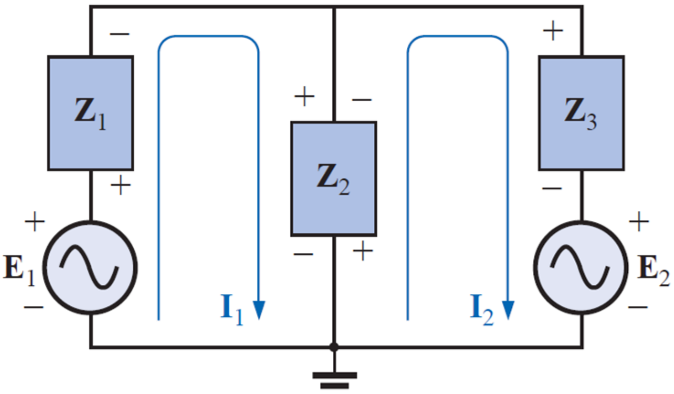
$\mathbf{Z}_1 = +j\,X_L = +j\,2\ \Omega$

$\mathbf{Z}_2 = R = 4\ \Omega$

$\mathbf{Z}_3 = -j\,X_C = -j\,1\ \Omega$

$\mathbf{E}_1 = 2\ \mathrm{V} \angle 0°$

$\mathbf{E}_2 = 6\ \mathrm{V} \angle 0°$



$$+\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{Z}_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$-\mathbf{Z}_2(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 = 0$$

or

$$\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2\mathbf{Z}_2 = 0$$
$$-\mathbf{I}_2\mathbf{Z}_2 + \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 = 0$$

so that

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1$$
$$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 = -\mathbf{E}_2$$

which are rewritten as

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1$$
$$-\mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) = -\mathbf{E}_2$$

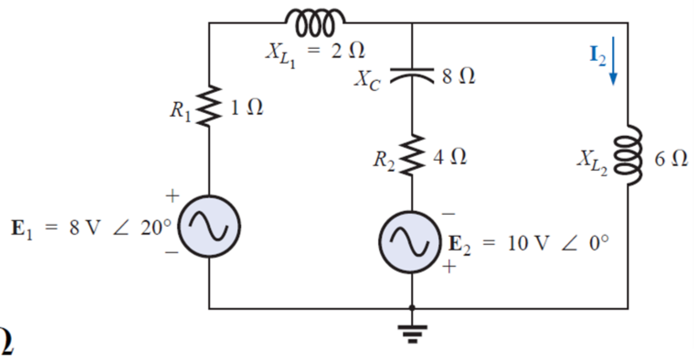
Using determinants, we obtain

$$\mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{E}_1 & -\mathbf{Z}_2 \\ -\mathbf{E}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}$$
$$= \frac{\mathbf{E}_1(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{E}_2(\mathbf{Z}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - (\mathbf{Z}_2)^2}$$
$$= \frac{(\mathbf{E}_1 - \mathbf{E}_2)\mathbf{Z}_2 + \mathbf{E}_1\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

Substituting numerical values yields

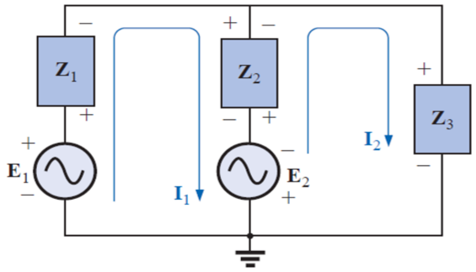
$$\mathbf{I}_1 = \frac{(2\ \mathrm{V} - 6\ \mathrm{V})(4\ \Omega) + (2\ \mathrm{V})(-j\,1\ \Omega)}{(+j\,2\ \Omega)(4\ \Omega) + (+j\,2\ \Omega)(-j\,2\ \Omega) + (4\ \Omega)(-j\,2\ \Omega)}$$
$$= \frac{-16 - j\,2}{j\,8 - j^2 2 - j\,4} = \frac{-16 - j\,2}{2 + j\,4} = \frac{16.12\ \mathrm{A} \angle -172.87°}{4.47 \angle 63.43°}$$
$$= \mathbf{3.61\ A \angle -236.30°} \quad \text{or} \quad \mathbf{3.61\ A \angle 123.70°}$$

Example:
Using mesh analysis, find the current I_2



Solution:

$$\mathbf{Z}_1 = R_1 + jX_{L_1} = 1\ \Omega + j\,2\ \Omega$$
$$\mathbf{Z}_2 = R_2 - jX_C = 4\ \Omega - j\,8\ \Omega$$
$$\mathbf{Z}_3 = +jX_{L_2} = +j\,6\ \Omega$$
$$\mathbf{E}_1 = 8\ \text{V}\ \angle 20^\circ$$
$$\mathbf{E}_2 = 10\ \text{V}\ \angle 0^\circ$$



$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1 + \mathbf{E}_2$$
$$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 = -\mathbf{E}_2$$

$$\begin{aligned}\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 \qquad\qquad &= \mathbf{E}_1 + \mathbf{E}_2\\ -\mathbf{I}_1\mathbf{Z}_2 \qquad\qquad + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) &= -\mathbf{E}_2\end{aligned}$$

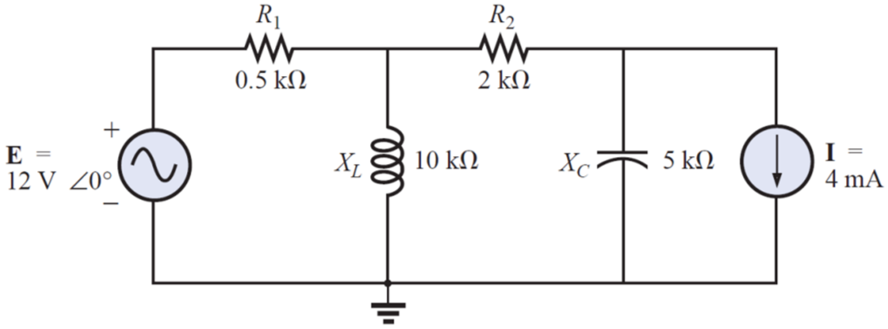
$$\mathbf{I}_2 = \frac{\begin{vmatrix}\mathbf{Z}_1 + \mathbf{Z}_2 & \mathbf{E}_1 + \mathbf{E}_2\\ -\mathbf{Z}_2 & -\mathbf{E}_2\end{vmatrix}}{\begin{vmatrix}\mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2\\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3\end{vmatrix}}$$
$$= \frac{-(\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{E}_2 + \mathbf{Z}_2(\mathbf{E}_1 + \mathbf{E}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{Z}_2^2}$$
$$= \frac{\mathbf{Z}_2\mathbf{E}_1 - \mathbf{Z}_1\mathbf{E}_2}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

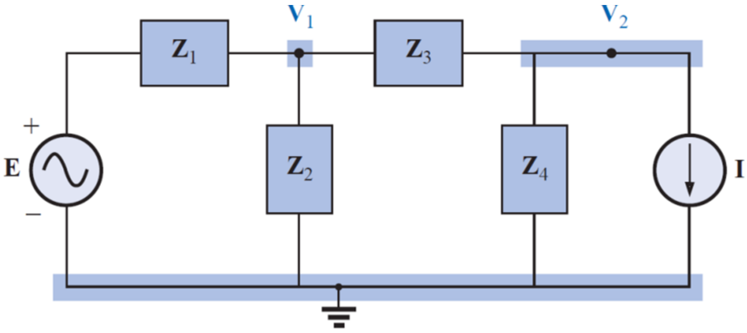
$$\mathbf{I}_2 = \frac{(4\ \Omega - j\,8\ \Omega)(8\ \text{V}\ \angle 20^\circ) - (1\ \Omega + j\,2\ \Omega)(10\ \text{V}\ \angle 0^\circ)}{(1\ \Omega + j\,2\ \Omega)(4\ \Omega - j\,8\ \Omega) + (1\ \Omega + j\,2\ \Omega)(+j\,6\ \Omega) + (4\ \Omega - j\,8\ \Omega)(+j\,6\ \Omega)}$$
$$= \frac{(4 - j\,8)(7.52 + j\,2.74) - (10 + j\,20)}{20 + (j\,6 - 12) + (j\,24 + 48)}$$
$$= \frac{(52.0 - j\,49.20) - (10 + j\,20)}{56 + j\,30} = \frac{42.0 - j\,69.20}{56 + j\,30} = \frac{80.95\ \text{A}\ \angle -58.74^\circ}{63.53\ \angle 28.18^\circ}$$
$$= \mathbf{1.27\ A\ \angle -86.92^\circ}$$

NODAL ANALYSIS

Example

Determine the voltage across the inductor for the network





For the application of Kirchhoff's current law to node $\mathbf{V}_1$:

$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$

$$0 = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

$$\frac{\mathbf{V}_1 - \mathbf{E}}{\mathbf{Z}_1} + \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_3} = 0$$

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right] = \frac{\mathbf{E}_1}{\mathbf{Z}_1}$$

For the application of Kirchhoff's current law to node $\mathbf{V}_2$:

$$0 = \mathbf{I}_3 + \mathbf{I}_4 + \mathbf{I}$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{\mathbf{Z}_3} + \frac{\mathbf{V}_2}{\mathbf{Z}_4} + \mathbf{I} = 0$$

Rearranging terms:

$$\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_4}\right] - \mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right] = -\mathbf{I}$$

Grouping equations:

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right] = \frac{\mathbf{E}}{\mathbf{Z}_1}$$

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_4}\right] = \mathbf{I}$$

$$\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{0.5\ \text{k}\Omega} + \frac{1}{j\ 10\ \text{k}\Omega} + \frac{1}{2\ \text{k}\Omega} = 2.5\ \text{mS}\ \angle -2.29^\circ$$

$$\frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_4} = \frac{1}{2\ \text{k}\Omega} + \frac{1}{-j\ 5\ \text{k}\Omega} = 0.539\ \text{mS}\ \angle 21.80^\circ$$

and

$$\mathbf{V}_1[2.5\ \text{mS}\ \angle -2.29^\circ] - \mathbf{V}_2[0.5\ \text{mS}\ \angle 0^\circ] = 24\ \text{mA}\ \angle 0^\circ$$

$$\mathbf{V}_1[0.5\ \text{mS}\ \angle 0^\circ] - \mathbf{V}_2[0.539\ \text{mS}\ \angle 21.80^\circ] = 4\ \text{mA}\ \angle 0^\circ$$

with

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 24 \text{ mA } \angle 0^\circ & -0.5 \text{ mS } \angle 0^\circ \\ 4 \text{ mA } \angle 0^\circ & -0.539 \text{ mS } \angle 21.80^\circ \end{vmatrix}}{\begin{vmatrix} 2.5 \text{ mS } \angle -2.29^\circ & -0.5 \text{ mS } \angle 0^\circ \\ 0.5 \text{ mS } \angle 0^\circ & -0.539 \text{ mS } \angle 21.80^\circ \end{vmatrix}}$$

$$= \frac{(24 \text{ mA } \angle 0^\circ)(-0.539 \text{ mS } \angle 21.80^\circ) + (0.5 \text{ mS } \angle 0^\circ)(4 \text{ mA } \angle 0^\circ)}{(2.5 \text{ mS } \angle -2.29^\circ)(-0.539 \text{ mS } \angle 21.80^\circ) + (0.5 \text{ mS } \angle 0^\circ)(0.5 \text{ mS } \angle 0^\circ)}$$

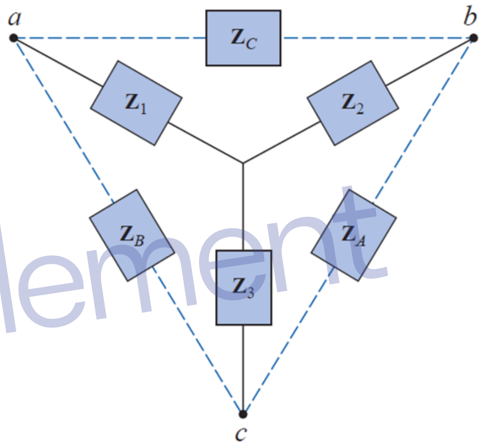
$$= \frac{-12.94 \times 10^{-6} \text{ V } \angle 21.80^\circ + 2 \times 10^{-6} \text{ V } \angle 0^\circ}{-1.348 \times 10^{-6} \angle 19.51^\circ + 0.25 \times 10^{-6} \angle 0^\circ}$$

$$= \frac{-(12.01 + j\,4.81) \times 10^{-6} \text{ V} + 2 \times 10^{-6} \text{ V}}{-(1.271 + j\,0.45) \times 10^{-6} + 0.25 \times 10^{-6}}$$

$$= \frac{-10.01 \text{ V} - j\,4.81 \text{ V}}{-1.021 - j\,0.45} = \frac{11.106 \text{ V } \angle -154.33^\circ}{1.116 \angle -156.21^\circ}$$

$$\mathbf{V}_1 = \mathbf{9.95 \text{ V } \angle 1.88^\circ}$$

Δ-Y, Y-Δ CONVERSIONS



$$\mathbf{Z}_1 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

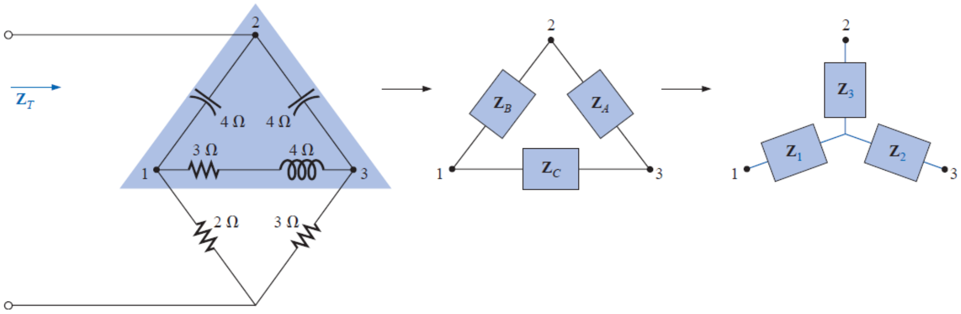
$$\mathbf{Z}_B = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2}$$

$$\mathbf{Z}_A = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1}$$

$$\mathbf{Z}_C = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_3}$$

Example:

Find the total impedance $\mathbf{Z}_T$ of the network



$$\mathbf{Z}_B = -j\,4 \qquad \mathbf{Z}_A = -j\,4 \qquad \mathbf{Z}_C = 3 + j\,4$$

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\,\Omega)(3\,\Omega + j\,4\,\Omega)}{(-j\,4\,\Omega) + (-j\,4\,\Omega) + (3\,\Omega + j\,4\,\Omega)}$$

$$= \frac{(4\,\angle -90^\circ)(5\,\angle 53.13^\circ)}{3 - j\,4} = \frac{20\,\angle -36.87^\circ}{5\,\angle -53.13^\circ}$$

$$= 4\,\Omega\,\angle 16.13^\circ = 3.84\,\Omega + j\,1.11\,\Omega$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\,\Omega)(3\,\Omega + j\,4\,\Omega)}{5\,\Omega\,\angle -53.13^\circ}$$

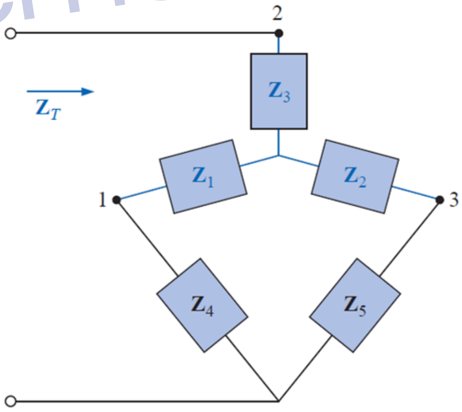
$$= 4\,\Omega\,\angle 16.13^\circ = 3.84\,\Omega + j\,1.11\,\Omega$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\,\Omega)(-j\,4\,\Omega)}{5\,\Omega\,\angle -53.13^\circ}$$

$$= \frac{16\,\Omega\,\angle -180^\circ}{5\,\angle -53.13^\circ} = 3.2\,\Omega\,\angle -126.87^\circ = -1.92\,\Omega - j\,2.56\,\Omega$$

Replace the Δ by the Y (Fig. 17.49):

$$\mathbf{Z}_1 = 3.84\,\Omega + j\,1.11\,\Omega \qquad \mathbf{Z}_2 = 3.84\,\Omega + j\,1.11\,\Omega$$
$$\mathbf{Z}_3 = -1.92\,\Omega - j\,2.56\,\Omega \qquad \mathbf{Z}_4 = 2\,\Omega$$
$$\mathbf{Z}_5 = 3\,\Omega$$



Impedances $\mathbf{Z}_1$ and $\mathbf{Z}_4$ are in series:

$$\mathbf{Z}_{T_1} = \mathbf{Z}_1 + \mathbf{Z}_4 = 3.84\,\Omega + j\,1.11\,\Omega + 2\,\Omega = 5.84\,\Omega + j\,1.11\,\Omega$$
$$= 5.94\,\Omega\,\angle 10.76^\circ$$

Impedances $\mathbf{Z}_2$ and $\mathbf{Z}_5$ are in series:

$$\mathbf{Z}_{T_2} = \mathbf{Z}_2 + \mathbf{Z}_5 = 3.84\,\Omega + j\,1.11\,\Omega + 3\,\Omega = 6.84\,\Omega + j\,1.11\,\Omega$$
$$= 6.93\,\Omega\,\angle 9.22^\circ$$

Impedances $\mathbf{Z}_{T_1}$ and $\mathbf{Z}_{T_2}$ are in parallel:

$$\mathbf{Z}_{T_3} = \frac{\mathbf{Z}_{T_1}\mathbf{Z}_{T_2}}{\mathbf{Z}_{T_1} + \mathbf{Z}_{T_2}} = \frac{(5.94\,\Omega\,\angle 10.76^\circ)(6.93\,\Omega\,\angle 9.22^\circ)}{5.84\,\Omega + j\,1.11\,\Omega + 6.84\,\Omega + j\,1.11\,\Omega}$$

$$= \frac{41.16\,\Omega\,\angle 19.98^\circ}{12.68 + j\,2.22} = \frac{41.16\,\Omega\,\angle 19.98^\circ}{12.87\,\angle 9.93^\circ} = 3.198\,\Omega\,\angle 10.05^\circ$$

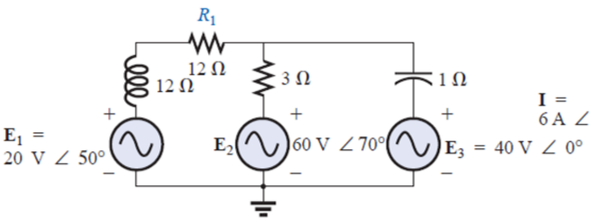
$$= 3.15\,\Omega + j\,0.56\,\Omega$$

Impedances $\mathbf{Z}_3$ and $\mathbf{Z}_{T_3}$ are in series. Therefore,

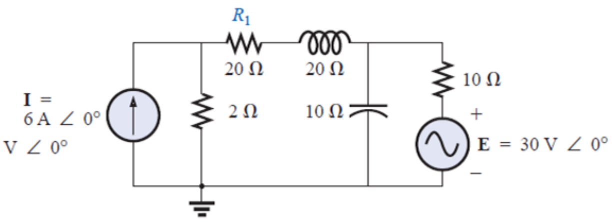
$$\mathbf{Z}_T = \mathbf{Z}_3 + \mathbf{Z}_{T_3} = -1.92\,\Omega - j\,2.56\,\Omega + 3.15\,\Omega + j\,0.56\,\Omega$$
$$= 1.23\,\Omega - j\,2.0\,\Omega = \mathbf{2.35\,\Omega\,\angle -58.41^\circ}$$

Tutorial

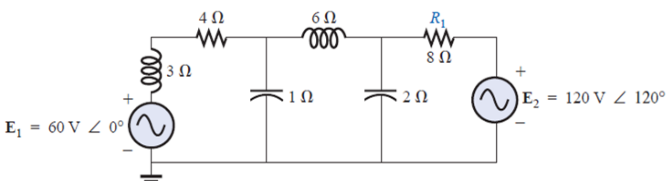
1-Write the mesh equations for the networks. Determine the current through the resistor R_1.

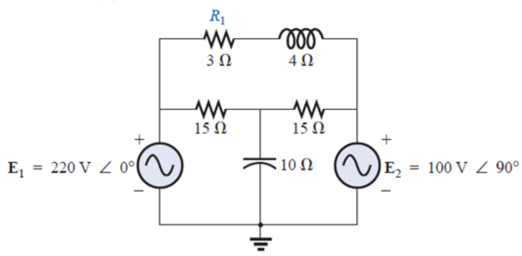


(a)

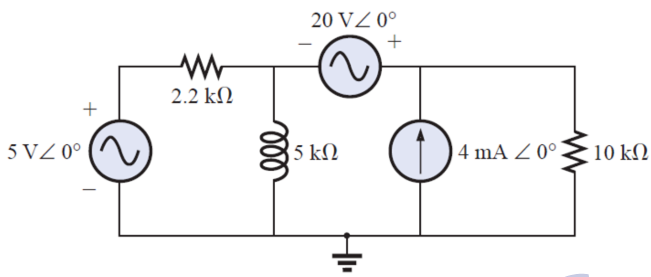


(b)

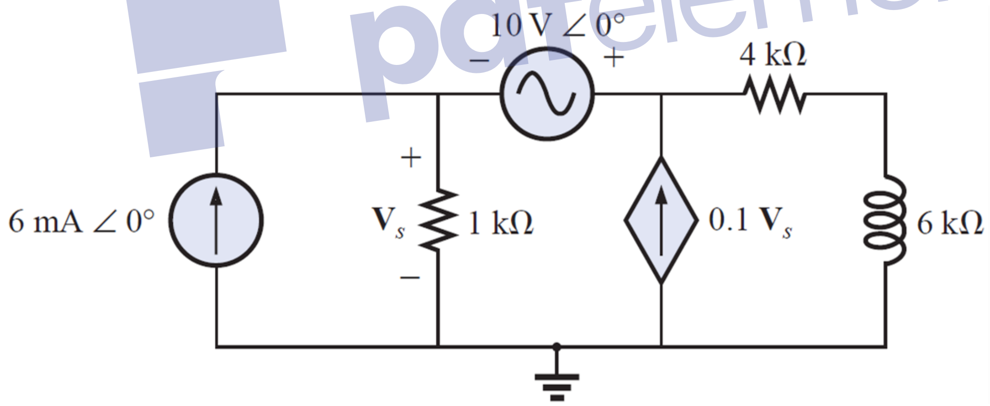




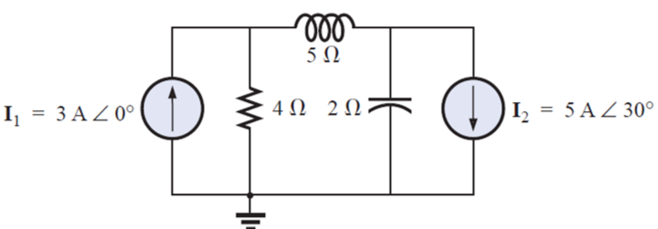
2-Write the mesh equations for the network, and determine the current through the 10 kΩ resistor.

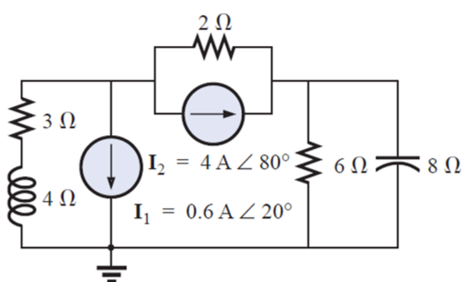


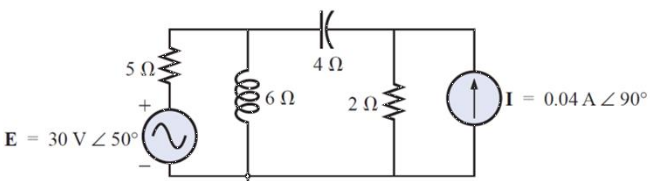
3-Write the mesh equations for the network, and determine the current through the inductive element.

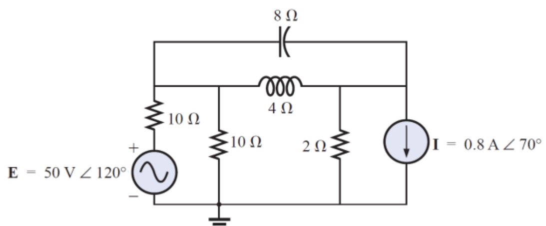


4- Determine the nodal voltages for the networks

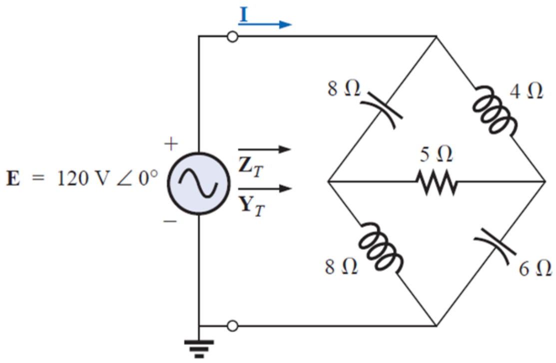


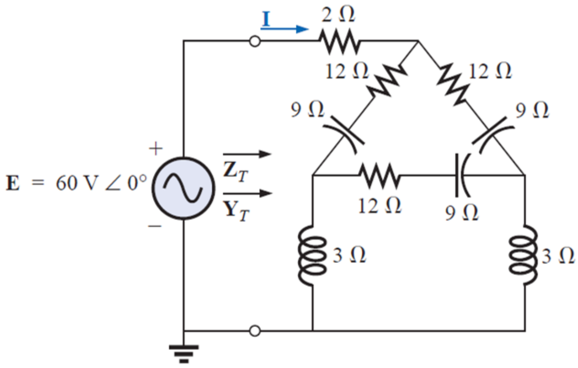






5-Determine the current **I** for the networks





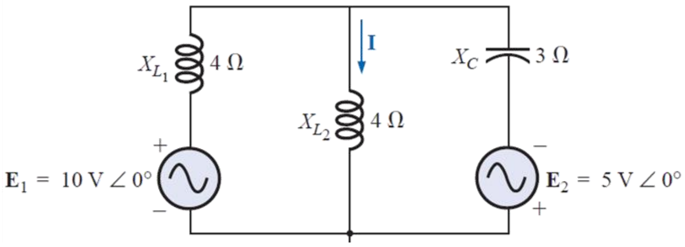
Network Theorems (ac)

SUPERPOSITION THEOREM

One of the most frequent applications of the superposition theorem is to electronic systems in which the dc and ac analyses are treated separately and the total solution is the sum of the two. It is an important application of the theorem because the impact of the reactive elements changes dramatically in response to the two types of independent sources.

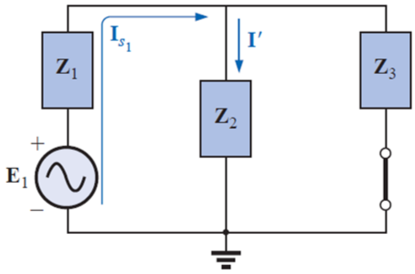
Example:

Using the superposition theorem, find the current **I** through the 4Ω reactance (X_{L2})



Solution:

$$\mathbf{Z}_1 = +j\,X_{L_1} = j\,4\,\Omega$$
$$\mathbf{Z}_2 = +j\,X_{L_2} = j\,4\,\Omega$$
$$\mathbf{Z}_3 = -j\,X_C = -j\,3\,\Omega$$



Considering the effects of the voltage source $\mathbf{E}_1$

$$\mathbf{Z}_{2\|3} = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(j\,4\,\Omega)(-j\,3\,\Omega)}{j\,4\,\Omega - j\,3\,\Omega} = \frac{12\,\Omega}{j} = -j\,12\,\Omega$$
$$= 12\,\Omega\,\angle -90^\circ$$

$$I_{s_1} = \frac{\mathbf{E}_1}{\mathbf{Z}_{2\|3} + \mathbf{Z}_1} = \frac{10\text{ V}\,\angle 0^\circ}{-j\,12\,\Omega + j\,4\,\Omega} = \frac{10\text{ V}\,\angle 0^\circ}{8\,\Omega\,\angle -90^\circ}$$
$$= 1.25\text{ A}\,\angle 90^\circ$$

and

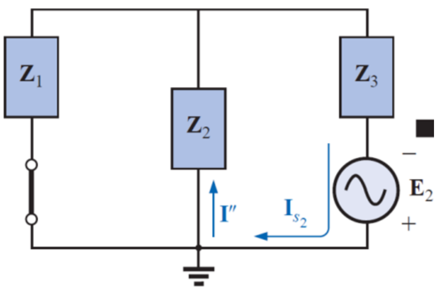
$$\mathbf{I}' = \frac{\mathbf{Z}_3\mathbf{I}_{s_1}}{\mathbf{Z}_2 + \mathbf{Z}_3} \qquad \text{(current divider rule)}$$
$$= \frac{(-j\,3\,\Omega)(j\,1.25\text{ A})}{j\,4\,\Omega - j\,3\,\Omega} = \frac{3.75\text{ A}}{j\,1} = 3.75\text{ A}\,\angle -90^\circ$$

Considering the effects of the voltage source $\mathbf{E}_2$, we have

$$\mathbf{Z}_{1\|2} = \frac{\mathbf{Z}_1}{N} = \frac{j\,4\,\Omega}{2} = j\,2\,\Omega$$

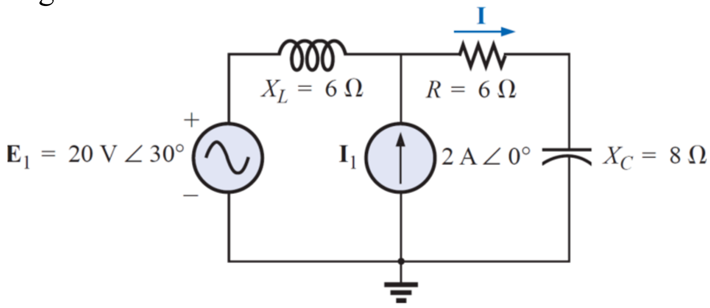
$$\mathbf{I}_{s_2} = \frac{\mathbf{E}_2}{\mathbf{Z}_{1\|2} + \mathbf{Z}_3} = \frac{5\text{ V}\,\angle 0^\circ}{j\,2\,\Omega - j\,3\,\Omega} = \frac{5\text{ V}\,\angle 0^\circ}{1\,\Omega\,\angle -90^\circ} = 5\text{ A}\,\angle 90^\circ$$

$$\mathbf{I}'' = \frac{\mathbf{I}_{s_2}}{2} = 2.5\text{ A}\,\angle 90^\circ$$



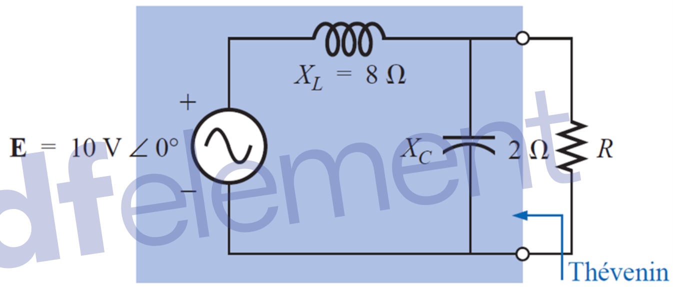
$$\mathbf{I} = \mathbf{I}' - \mathbf{I}''$$
$$= 3.75\text{ A}\angle -90^\circ - 2.50\text{ A}\angle 90^\circ = -j\,3.75\text{ A} - j\,2.50\text{ A}$$
$$= -j\,6.25\text{ A}$$
$$\mathbf{I} = 6.25\text{ A}\angle -90^\circ$$

Example:
Using superposition, find the current **I** through the 6Ω resistor.



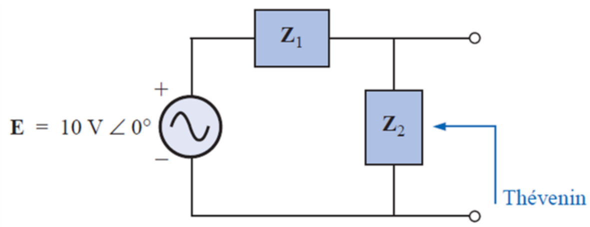
THE´VENIN'S THEOREM

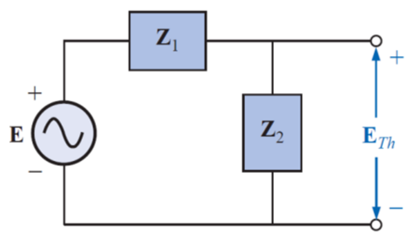
Example:
Find the Thévenin equivalent circuit for the network external to resistor R

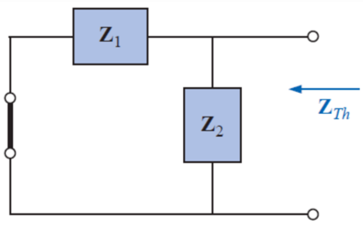


Solution:

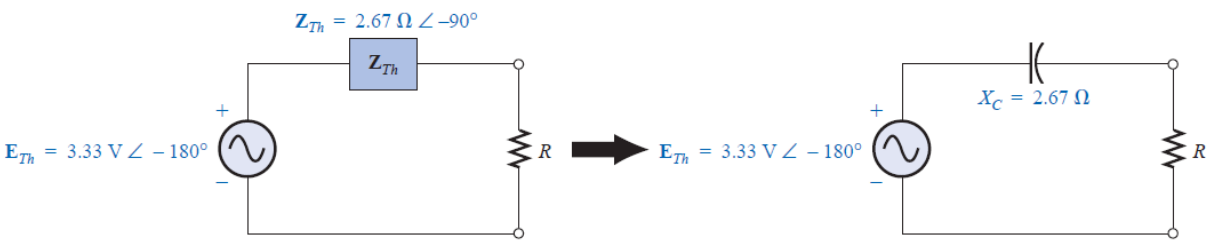
$Z_1 = jX_L = j8\ \Omega$
$Z_2 = -jX_C = -j2\ \Omega$
$$Z_{th} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{16}{j6} = 2.67\angle -90\ \ \Omega$$
$$E_{th} = \frac{E Z_2}{Z_1 + Z_2} = \frac{-j20}{j6} = 3.33\angle -180\ V$$





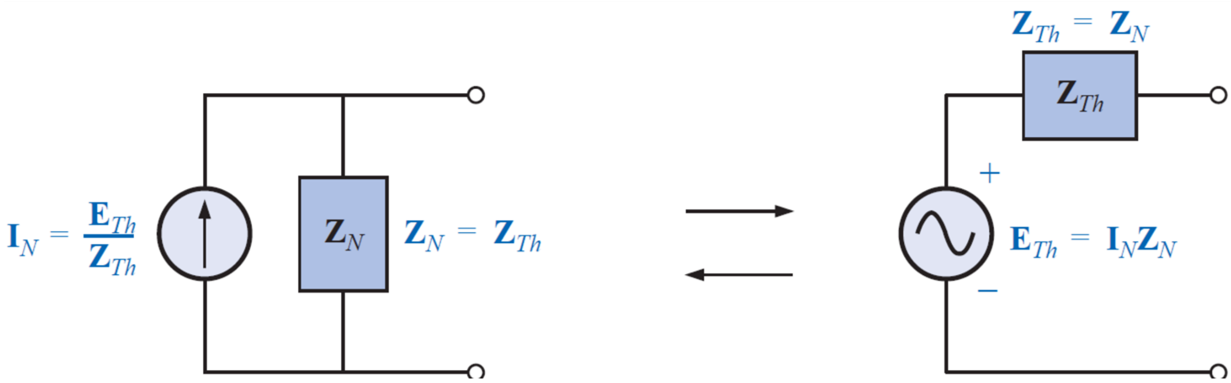


The Thévenin equivalent circuit is shown below



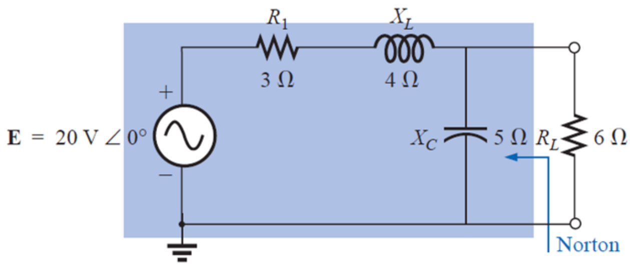
NORTON'S THEOREM

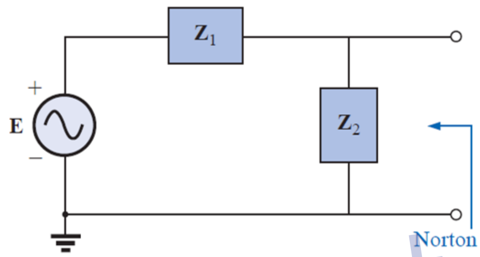
The Norton and Thévenin equivalent circuits can be found from each other by using the source transformation shown in figurer below.



Example:

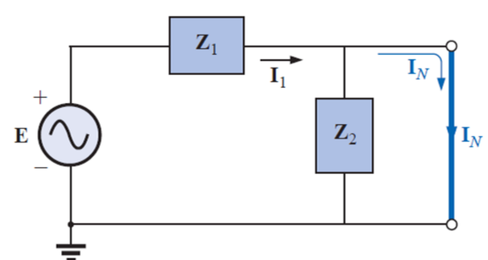
Determine the Norton equivalent circuit for the network external to the 6Ω resistor

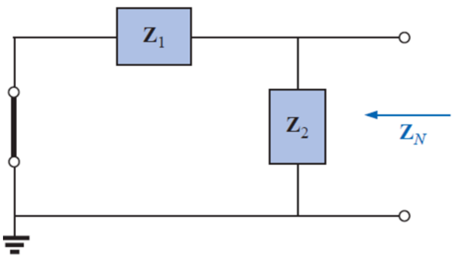




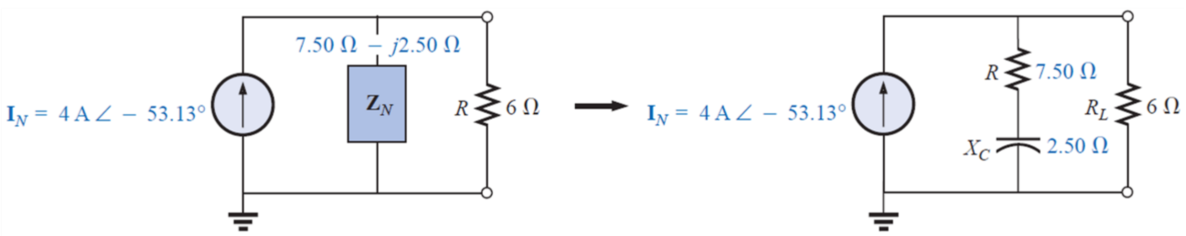
Solution:

$$Z_1 = R_1 + jX_L = 3 + j4 = 5\angle 53.1\ \Omega$$
$$Z_2 = -jX_C = -j5\ \Omega$$
$$Z_N = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{5\angle 53.1 \times 5\angle -90}{3 + j4 - j5} = 7.91\angle -18.44 = \mathbf{7.50 - j\,2.50}\Omega$$
$$I_N = \frac{E}{Z_1} = \frac{20}{5\angle 53.1} = 4\angle -53.1\ A$$





The Norton equivalent circuit is shown in figure below



MAXIMUM POWER TRANSFER THEOREM

When applied to ac circuits, the **maximum power transfer theorem** states that ***maximum power will be delivered to a load when the load impedance is the conjugate of the Thévenin impedance across its terminals.***

That is, for maximum power transfer to the load,

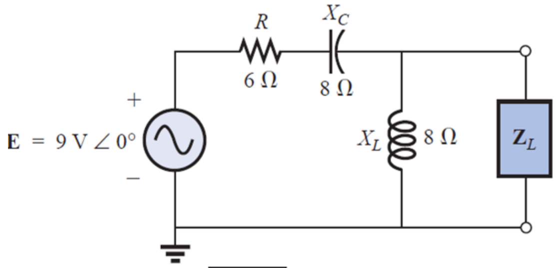
$Z_L = Z_{th}^{*} \qquad \theta_L = -\theta_{th}$

Therefore

$Z_T = R \mp jX + R \pm jX = 2R$

Example:

Find the load impedance for maximum power to the load, and find the maximum power.

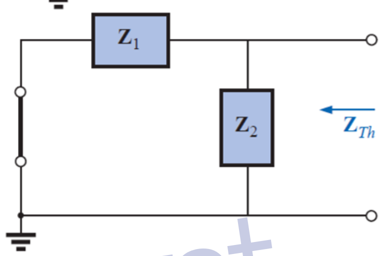


$Z_1 = 6 - j8 = 10\angle -53.1$

$Z_2 = j8$

$$Z_{th} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{10\angle -53.1 \times 8\angle 90}{6 - j8 + j8} = 13.33\angle 36.87°$$

$= 10.66 + j8$

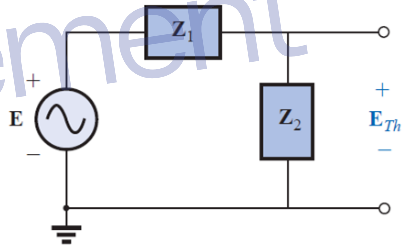


$Z_L = Z_{th}^{*} = 13.33\angle -36.87° = 10.66 - j8$

$$E_{th} = \frac{E Z_2}{Z_1 + Z_2} = \frac{9\angle 0 \times 8\angle 90}{6 - j8 + j8} = 12\angle 90° \text{ V}$$

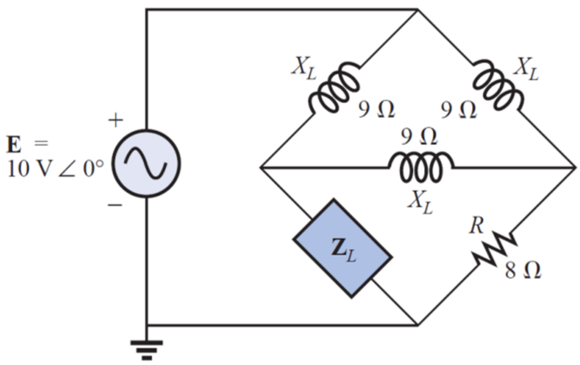
Then

$$P_{max} = \frac{E_{th}^2}{4R} = \frac{12^2}{4 \times 10.66} = 3.38\ w$$



Example:

Find the load impedance for maximum power to the load, and find the maximum power



Power (ac)

For any system, the power delivered to a load at any instant is defined by the product of the applied voltage and the resulting current; that is,

$$p = vi$$

In this case, since *v* and *i* are sinusoidal quantities, let us establish a general case where

$$v = V_m \sin(\omega t + \theta)$$
$$i = I_m \sin \omega t$$

Substituting the above equations for *v* and *i* into the power equation will result in

$$p = vi = V_m I_m \sin \omega t \sin(\omega t + \theta)$$
$$p = VIcos\theta(1 - cos2\omega t) + VI\sin\theta \sin 2\omega t$$

RESISTIVE CIRCUIT

For a purely resistive circuit , *v* and *i* are in phase,

$$p_R = VIcos0(1 - cos2\omega t) + VI\sin 0 \sin 2\omega t$$
$$p_R = VI(1 - cos2\omega t)$$

APPARENT POWER

$$S = VI \quad volt - amperes, VA$$
$$p = Scos\theta = SF_p$$

INDUCTIVE CIRCUIT AND REACTIVE POWER

For a purely inductive circuit , *v* leads *i* by 90°,

$$p_L = VIcos90(1 - cos2\omega t) + VI\sin 90 \sin 2\omega t$$
$$p_L = VI\sin 2\omega t$$

In general, the reactive power associated with any circuit is defined to be $VI\sin\theta$,
The symbol for reactive power is *Q*, and its unit of measure is the *volt-ampere reactive* (VAR).

$$Q = VI\sin\theta \quad \text{volt-ampere reactive, VAR}$$

For the inductor

$$Q_L = VI$$
$$F_p = cos\theta = cos90 = 0$$

CAPACITIVE CIRCUIT

$$p_C = VIcos(-90)(1 - cos2\omega t) + VI\sin(-90)\sin 2\omega t$$
$$p_C = -VI\sin 2\omega t$$
$$Q_C = VI \qquad \text{VAR}$$
$$F_p = cos\theta = cos90 = 0$$

THE POWER TRIANGLE

The three quantities **average power, apparent power,** and **reactive power** can be related in the vector domain by

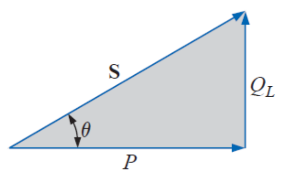
$$S = P + Q$$

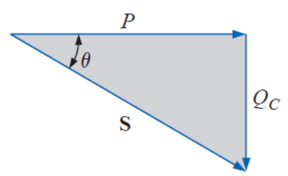
For an inductive load, the *phasor power* **S,** as it is often called, is defined by

$$S = P + jQ_L$$

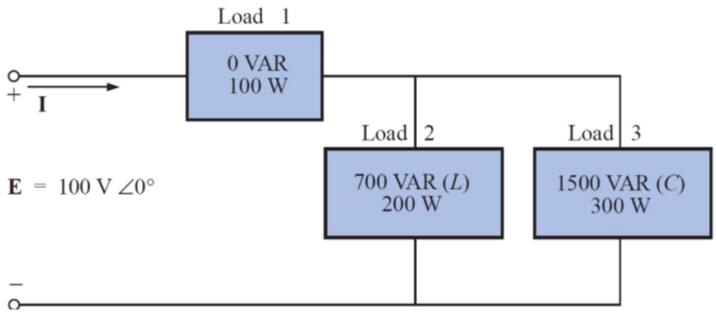
For a capacitive load, the phasor power **S** is defined by

$$S = P - jQ_C$$

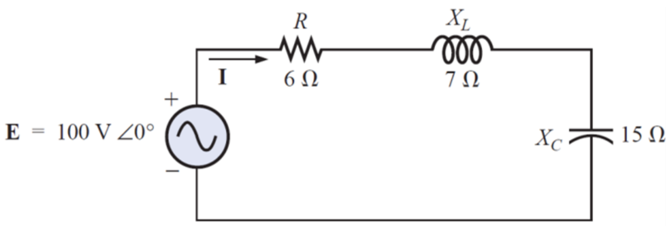




Example:
Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p of the network. Draw the power triangle and find the current in phasor form.



Example:
Find the total number of watts, volt-amperes reactive, and volt amperes, and the power factor F_p for the network and sketch the power triangle.



Example:
An electrical device is rated 5 kVA, 100 V at a 0.6 power-factor lag. What is the impedance of the device in rectangular coordinates?

POWER-FACTOR CORRECTION

The process of introducing reactive elements to bring the power factor closer to unity is called **power-factor correction.** Since most loads are inductive, the process normally involves introducing elements with capacitive terminal characteristics having the sole purpose of improving the power factor.

Example:
A 5-hp motor with a 0.6 lagging power factor and an efficiency of 92% is connected to a 208-V, 60-Hz supply.
a. Establish the power triangle for the load.
b. Determine the power-factor capacitor that must be placed in parallel with the load to raise the power factor to unity.
c. Determine the change in supply current from the uncompensated to the compensated system.

Example:
a. A small industrial plant has a 10-kW heating load and a 20-kVA inductive load due to a bank of induction motors. The heating elements are considered purely resistive ($Fp = 1$), and the induction motors have a lagging power factor of 0.7. If the supply is 1000 V at 60 Hz, determine the capacitive element required to raise the power factor to 0.95.
b. Compare the levels of current drawn from the supply.

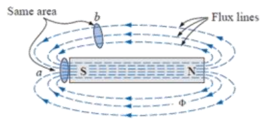
Magnetic Circuits

INTRODUCTION

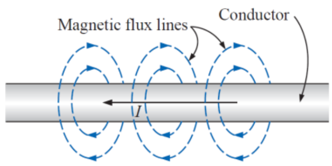
Magnetism plays an integral part in almost every electrical device used today in industry, research, or the home. Generators, motors, transformers, circuit breakers, televisions, computers, tape recorders, and telephones all employ magnetic effects to perform a variety of important tasks.

MAGNETIC FIELDS

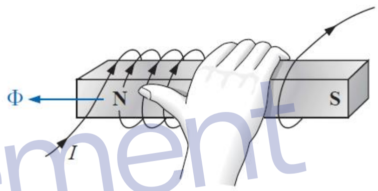
In the region surrounding a permanent magnet there exists a magnetic field, which can be represented by **magnetic flux lines** similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in continuous loops, as shown in Figure below.
The symbol for magnetic flux is the Greek letter Φ (phi).



The magnetic flux lines radiate from the north pole to the south pole, returning to the north pole through the metallic bar.



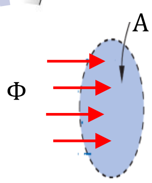
A magnetic field is present around every wire that carries an electric current. The direction of the magnetic flux lines can be found simply by placing the thumb of the right hand in the direction of conventional current flow and noting the direction of the fingers. (This method is commonly called the right-hand rule.)
In the SI system of units, magnetic flux is measured in *webers*. The number of flux lines per unit area is called the **flux density,** is denoted by the capital letter *B*, and is measured in *teslas*. Its magnitude is determined by the following equation:



$$B = \frac{\Phi}{A}$$

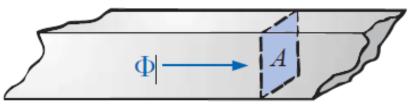
B = teslas (T)
Φ = webers (Wb)
A = square meters (m^2)

where Φ is the number of flux lines passing through the area *A*.



Example
Determine the flux density

$$B = \frac{\Phi}{A} = \frac{6\times10^{-5}}{1.2\times10^{-3}} = 5\times10^{-2}T$$



$\Phi = 6 \times 10^{-5}$ Wb
$A = 1.2 \times 10^{-3}$ m^2

PERMEABILITY

If cores of different materials with the same physical dimensions are used in the electromagnet described in Section 11.2, the strength of the magnet will vary in accordance with the core used. This variation in strength is due to the greater or lesser number of flux lines passing through the core. Materials in which flux lines can readily be set up are said to be *magnetic* and to have *high permeability*. The **permeability** (μ) of a material, therefore, is a measure of the ease with which magnetic flux lines can be established in the material. It is similar in many respects to conductivity in electric circuits. The permeability of free space μ_0 (vacuum) is

$$\mu_0 = 4\pi\times10^{-7}\frac{W}{A.m}$$

The ratio of the permeability of a material to that of free space is called its **relative permeability;** that is,

$$\mu_r = \frac{\mu}{\mu_0}$$

RELUCTANCE

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho\frac{l}{A}\ \Omega$$

The reluctance of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\mathcal{R} = \frac{l}{\mu A} \qquad At/Wb$$

Where $\mathcal{R}$ is the reluctance, l is the length of the magnetic path, and A is the cross-sectional area.

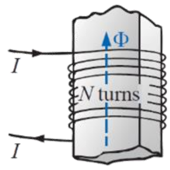
OHM'S LAW FOR MAGNETIC CIRCUITS

For magnetic circuits, the effect desired is the flux Φ. The cause is the **magnetomotive force (mmf)** $\mathcal{F}$ which is the external force (or "pressure") required to set up the magnetic flux lines within the magnetic material. The opposition to the setting up of the flux Φ is the reluctance $\mathcal{R}$.
Substituting, we have

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}}$$

The magnetomotive force $\mathcal{F}$ is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire.

$$\mathcal{F} = NI \text{ (ampere-turns, At)}$$



MAGNETIZING FORCE

The magnetomotive force per unit length is called the **magnetizing force** (H).

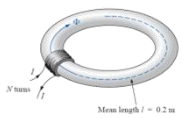
$$H = \frac{\mathcal{F}}{l} \quad \text{(At/m)}$$

Substituting for the magnetomotive force will result in

$$H = \frac{NI}{l} \quad \text{(At/m)}$$

Example:
Determine the magnetizing force for the following figure if N=20 and I=2A.
Sol

$$H = \frac{NI}{l} = \frac{40}{0.2} = 200 \quad \text{(At/m)}$$



The flux density and the magnetizing force are related by the following equation:

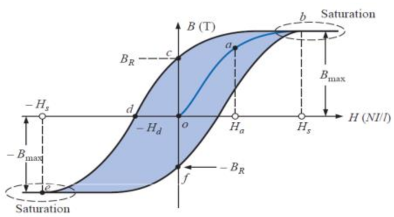
$$B = \mu H$$

HYSTERESIS

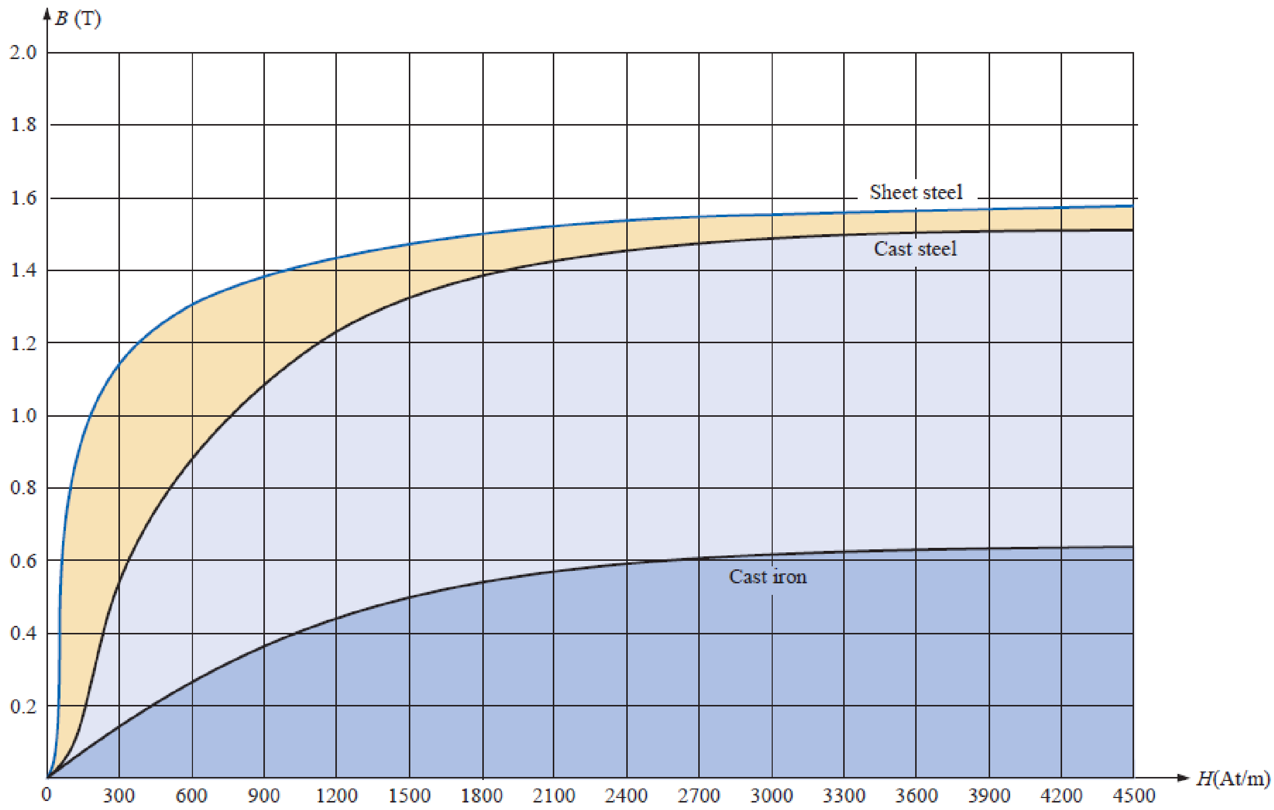
A curve of the flux density B versus the magnetizing force H of a material is of particular importance to the engineer. Curves of this type can usually be found in manuals, descriptive pamphlets, and brochures published by manufacturers of magnetic materials. A typical B-H curve for a ferromagnetic material such as steel can be derived using the following setups.
The core is initially unmagnetized and the current I =0. If the current I is increased to some value above zero, the magnetizing force H will increase to a value determined by

$$H\uparrow = \frac{NI\uparrow}{l}$$

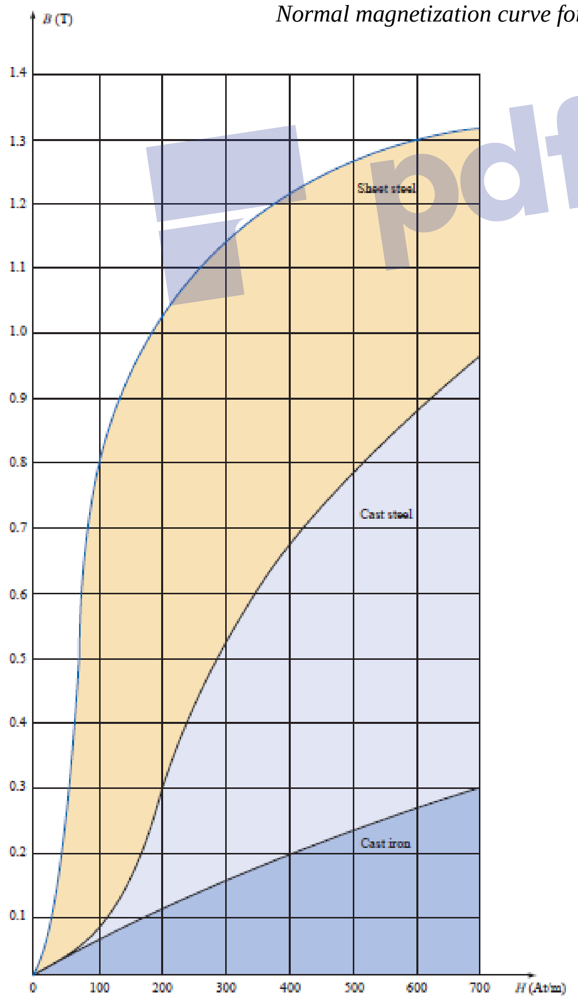
The flux Φ and the flux density B will also increase with the current I (or H).



Hysteresis curve.



Normal magnetization curve for three ferromagnetic materials



low magnetizing force region for three ferromagnetic materials

AMPÈRE'S CIRCUITAL LAW

	Electric Circuits	Magnetic Circuits
Cause	E	$\mathscr{F}$
Effect	I	Φ
Opposition	R	$\mathscr{R}$

If we apply the "cause" analogy to Kirchhoff's voltage law $\Sigma_{\circlearrowright} V = 0$, we obtain the following:

$$\Sigma_{\circlearrowright} \mathscr{F} = 0$$

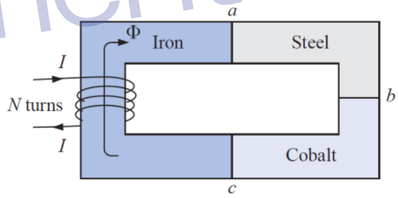
which, in words, states that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop.
This equation referred to as **Ampère's circuital law.** When it is applied to magnetic circuits, sources of mmf are expressed as

$$\mathscr{F} = NI \text{ (ampere-turns, At)}$$

And

$$\mathscr{F} = Hl \text{ (ampere-turns, At)}$$

Example:
Consider the magnetic circuit appearing in Figure below constructed of three different ferromagnetic materials.

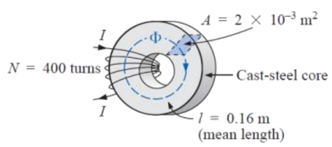


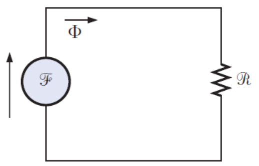
Solutions.
Applying Ampère's circuital law, we have

$$\Sigma_{\circlearrowright} \mathscr{F} = 0$$

$$NI - H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca} = 0$$
$$NI = H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}$$

Example
For the series magnetic circuit:
a. Find the value of I required to develop a magnetic flux of $\Phi = 4 \times 10^{-4}$ Wb.
b. Determine μ and μ_r for the material under these conditions.





Solutions:

$$B = \frac{\Phi}{A} = \frac{4\times10^{-4}}{2\times10^{-3}} = 0.2 \text{ T}$$

Using the B-H curves, we can determine the magnetizing force H:
H (cast steel) = 170 At/m
Applying Ampère's circuital law yields

$$NI = Hl$$

$$I = \frac{Hl}{N} = \frac{170\times0.16}{400} = 68\ mA$$

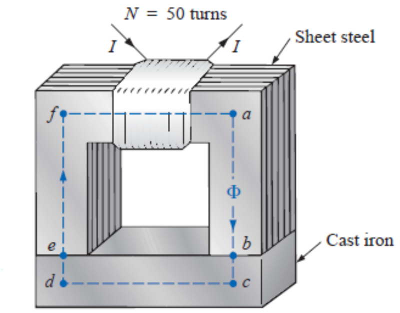
b. The permeability of the material can be found as

$$\mu = \frac{B}{H} = \frac{0.2}{170} = 1.176 \times 10^{-3} \text{ Wb/A.m}$$

and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_0} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = 935.83$$

Example:
The electromagnet of Figure below has picked up a section of cast iron. Determine the current I required to establish the indicated flux in the core, if $l_{ab} = l_{cd} = l_{ef} = l_{fa} = 101.6 \times 10^{-3}\ m$, $l_{bc} = l_{de} = 12.7 \times 10^{-3}\ m$, $\Phi = 3.5 \times 10^{-4}\ T$ and $A = 6.452 \times 10^{-4}\ m^2$



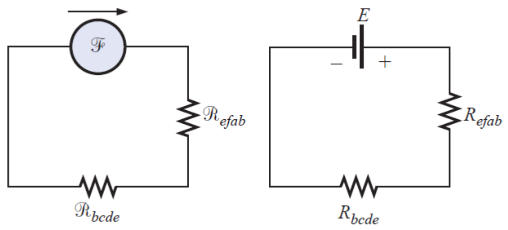
Solution:
The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{3.5\times10^{-4}}{6.452\times10^{-4}} = 0.542\ T$$

and the magnetizing force is
H (sheet steel) = 70 At/m
H (cast iron) = 1600 At/m
Determining Hl for each section yields

$$l_{efab} = l_{ef} + l_{fa} + l_{ab} = 3 \times 101.6 \times 10^{-3} = 304.8 \times 10^{-3}\ m$$
$$l_{bcde} = l_{bc} + l_{cd} + l_{de} = 101.6 \times 10^{-3} + 2 \times 12.7 \times 10^{-3} = 127 \times 10^{-3}\ m$$
$$H_{efab} l_{efab} = 70 \times 304.8 \times 10^{-3} = 21.34\ At$$
$$H_{bcde} l_{bcde} = 1600 \times 127 \times 10^{-3} = 203.2\ At$$

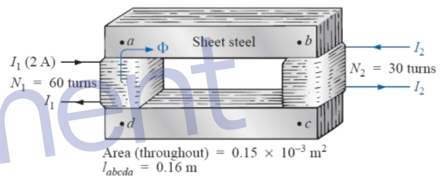
The magnetic circuit equivalent and the electric circuit analogy for the system



Applying Ampère's circuital law,

$$NI = H_{efab} l_{efab} + H_{bcde} l_{bcde} = 21.34 + 203.2 = 224.54$$
$$I = \frac{224.54}{50} = 4.49\ A$$

Example:
Determine the secondary current I_2 for the transformer of Figure below if the resultant clockwise flux in the core is $1.5 \times 10^{-5}\ Wb$

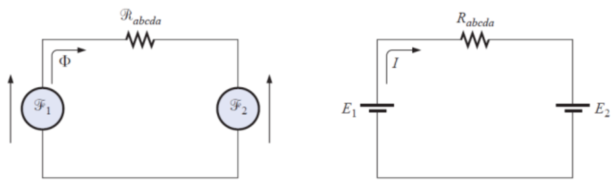


Solution:

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{1.5\times10^{-5}}{0.15\times10^{-3}} = 0.1\ T$$

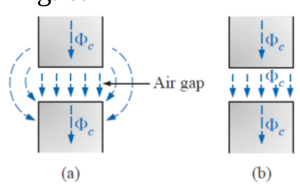
and the magnetizing force is
H (sheet steel) = 20 At/m
Applying Ampère's circuital law,



$$N_1 I_1 - N_2 I_2 = H l_{abcda}$$
$$60 \times 2 - 30 I_2 = 20 \times 0.16$$
$$I_2 = \frac{120 - 3.2}{30} = 3.89\ A$$

AIR GAPS

The spreading of the flux lines outside the common area of the core for the air gap in Fig. a is known as *fringing*. For our purposes, we shall neglect this effect and assume the flux distribution to be as in Fig. b.



The flux density of the air gap in is given by

$$B_g = \frac{\Phi_g}{A_g}$$

where, for our purposes,

$$\Phi_g = \Phi_{core}$$

And

$$A_g = A_{core}$$

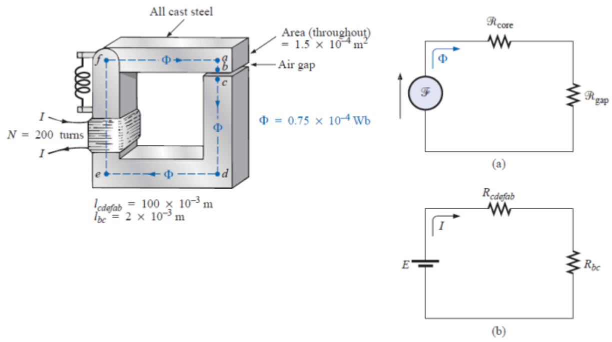
For most practical applications, the permeability of air is taken to be equal to that of free space. The magnetizing force of the air gap is then determined by

$$H_g = \frac{B_g}{\mu_0}$$

and the mmf drop across the air gap is equal to *Hglg*. An equation for *Hg* is as follows:

$$H_g = \frac{B_g}{\mu_0} = \frac{B_g}{4\pi \times 10^{-7}} = 7.96 \times 10^5 B_g \text{ (At/m)}$$

Example:
Find the value of I required to establish a magnetic flux of $\Phi = 0.75 \times 10^{-4}$ Wb in the series magnetic circuit of following Figure.



Solution:

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{7.5\times10^{-4}}{1.5\times10^{-4}} = 0.5\ T$$

From the *B-H* curves,
H (cast steel) = 280 At/m

$$H_g = 7.96 \times 10^5 B_g = 7.96 \times 10^5 \times 0.5 = 3.98 \times 10^5$$

Applying Ampère's circuital law,

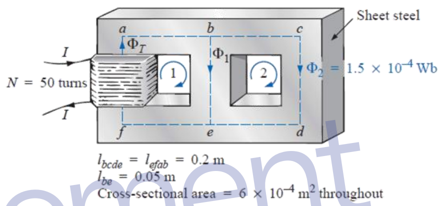
$$NI - Hl_{abcda} - H_g l_g = 0$$

$$I = \frac{280\times100\times10^{-3} + H_g l_g}{N} = \frac{280\times100\times10^{-3} + 3.98\times10^5\times10^{-3}}{200} = 4.12\ A$$

SERIES-PARALLEL MAGNETIC CIRCUITS

EXAMPLE

Determine the current I required to establish a flux of 1.5×10^{-4} Wb in the section of the core



Solution:

The equivalent magnetic circuit and the electric circuit analogy.
We have
The flux density for each section is

$$B_2 = \frac{\Phi_2}{A} = \frac{1.5\times10^{-4}}{6\times10^{-4}} = 0.25\ T$$

From the *B-H* curves,
H_{bcde} (sheet steel) = 40 At/m
Applying Ampère's circuital law around loop 2

$$\Sigma_{\circlearrowright} \mathcal{F} = 0$$

$$H_{be} l_{be} - H_{bcde} l_{bcde} = 0$$

$$H_{be} = \frac{H_{bcde} l_{bcde}}{l_{be}} = \frac{40\times0.2}{0.05} = 160\ At/m$$

From the *B-H* curves,

$$B_1 = 0.97\ T$$

$$\Phi_1 = B_1 A = 0.97 \times 6\times10^{-4} = 5.82\times10^{-4}\ Wb$$

The total flux density can be expressed as

$$\Phi_T = \Phi_1 + \Phi_2 = 5.82\times10^{-4} + 1.5\times10^{-4} = 7.32\times10^{-4}\ Wb$$

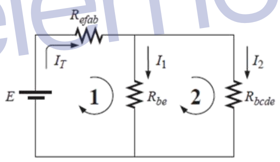
$$B_T = \frac{\Phi_T}{A} = \frac{7.32\times10^{-4}}{6\times10^{-4}} = 1.22\ T$$

From the *B-H* curves,
H_{efab} (sheet steel) = 400 At/m
Applying Ampère's circuital law around loop 1

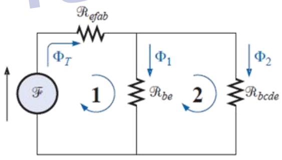
$$\Sigma_{\circlearrowright} \mathcal{F} = 0$$

$$NI - H_{be} l_{be} - H_{efab} l_{efab} = 0$$

$$I = \frac{160\times0.05 + 200\times0.2}{50} = 1.78\ A$$



Electric circuit analogy for the series parallel system



Magnetic circuit eauivalent

To demonstrate that m is sensitive to the magnetizing force *H*, the permeability of each section is determined as follows.
For section *bcde*,

$$\mu = \frac{B}{H} = \frac{0.25}{40} = 6.25\times10^{-3}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{6.25\times10^{-3}}{4\pi\times10^{-7}} = 4972.2$$

For section *be*,

$$\mu = \frac{B}{H} = \frac{0.97}{160} = 6.06\times10^{-3}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{6.06\times10^{-3}}{4\pi \times 10^{-7}} = 4821$$

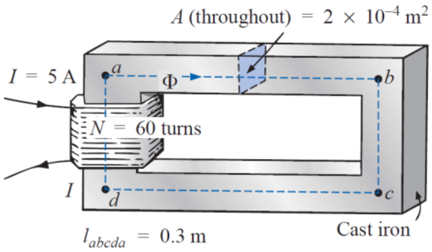
For section *be*,

$$\mu = \frac{B}{H} = \frac{1.22}{40} = 3.05 \times 10^{-3}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{3.05\times10^{-3}}{4\pi \times 10^{-7}} = 2426.41$$

Example:

Calculate the magnetic flux Φ for the magnetic circuit shown below:



Solution:

By Ampère's circuital law,

$$\Sigma_{\circlearrowright} \mathscr{F} = 0$$

$$NI - H_{abcda} l_{abcda} = 0$$

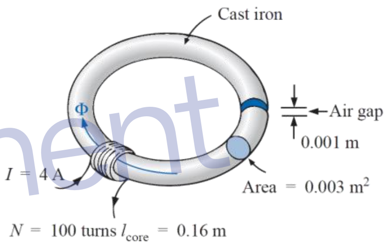
$$H_{abcda} = \frac{NI}{l_{abcda}} = \frac{5 \times 60}{0.3} = 1000 \text{ At/m}$$

B (cast iron from Figure)=0.39 T

$$\Phi = \text{BA} = 0.39 \times 2 \times 10^{-4} = 0.78 \times 10^{-4}\ Wb$$

Example:

Find the magnetic flux Φ for the series magnetic circuit of Figure below for the specified impressed mmf.



Solution:

Assuming that the total impressed mmf *NI* is across the air gap,

$$NI - H_g l_g$$

$$H_g = \frac{NI}{l_g} = \frac{4 \times 100}{0.001} = 4 \times 10^5\ At/m$$

$$B_g = \mu_0 H_g = 4\pi \times 10^{-7} \times 4 \times 10^5 = 0.503\ T$$

$$\Phi_g = \Phi_{core} = B_g A = 0.503 \times 0.003 = 1.51 \times 10^{-3}\ Wb$$

$$H_{core}\ (cast\ iron\ from\ B - H\ curve) = 1500\ At/m$$

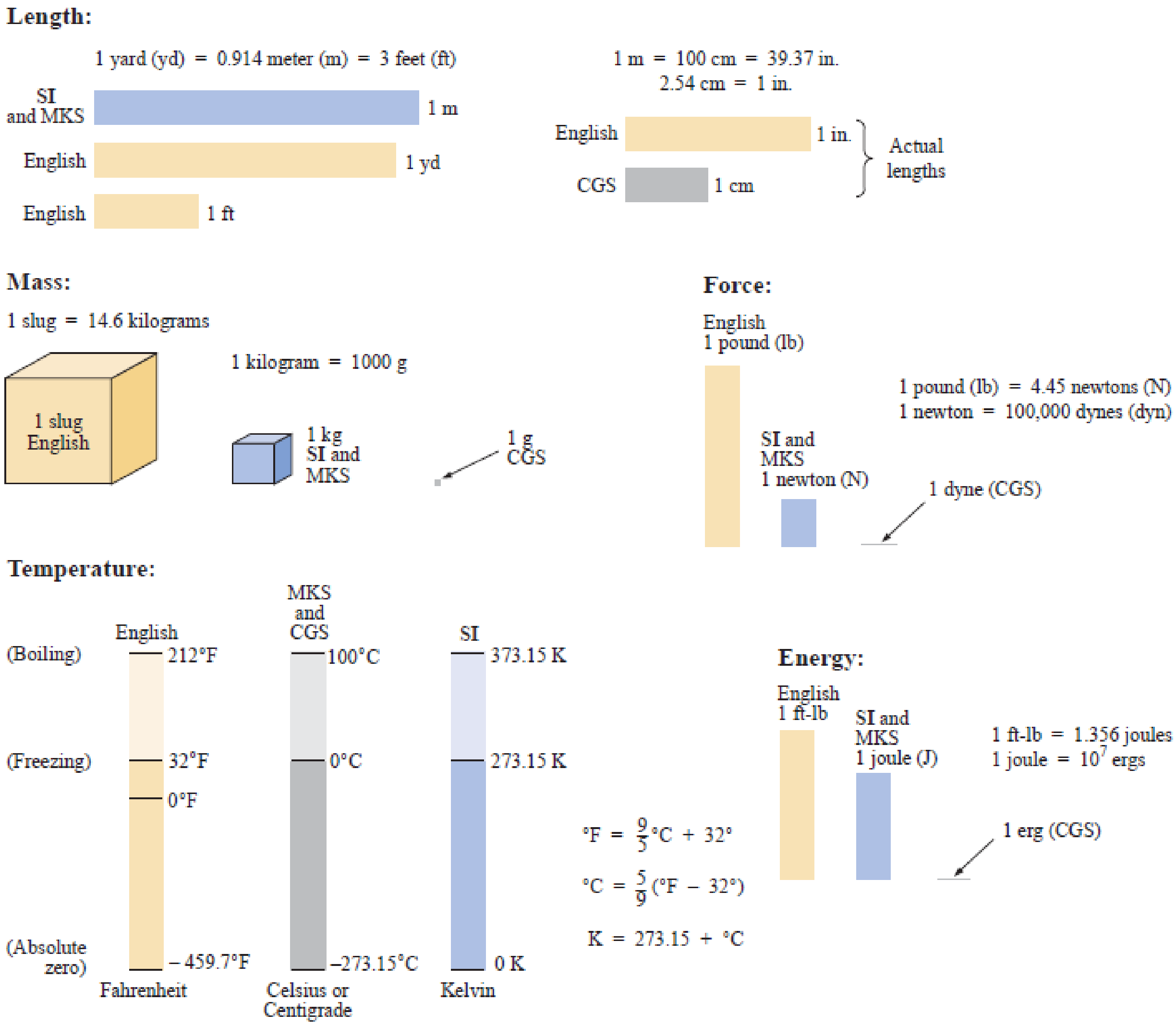
1.1 SYSTEMS OF UNITS

In the past, the *systems of units* most commonly used were the English and metric, as outlined in Table below. Note that while the English system is based on a single standard, the metric is subdivided into two interrelated standards: the MKS and the CGS.

Comparison of the English and metric systems of units.

English	Metric		
	MKS	CGS	SI
Length: Yard (yd) (0.914 m)	Meter (m) (39.37 in.) (100 cm)	Centimeter (cm) (2.54 cm = 1 in.)	**Meter (m)**
Mass: **Slug** (14.6 kg)	Kilogram (kg) (1000 g)	Gram (g)	**Kilogram (kg)**
Force: **Pound** (lb) (4.45 N)	Newton (N) (100,000 dynes)	Dyne	**Newton (N)**
Temperature: Fahrenheit (°F) $\left(= \frac{9}{5}°C + 32\right)$	Celsius or Centigrade (°C) $\left(= \frac{5}{9}(°F - 32)\right)$	Centigrade (°C)	**Kelvin (K)** K = 273.15 + °C
Energy: Foot-pound (ft-lb) (1.356 joules)	Newton-meter (N•m) or joule (J) (0.7376 ft-lb)	Dyne-centimeter or erg (1 joule = 10^7 ergs)	**Joule (J)**
Time: Second (s)	Second (s)	Second (s)	**Second (s)**

The International Bureau of Weights and Measures located at Sevres, France, has been the host for the General Conference of Weights and Measures, and attended by representatives from all nations of the world. In 1960, the General Conference adopted a system called Le Systems International unites (International System of Units), which has the international abbreviation **SI.** Since then, it has been adopted by the Institute of Electrical and Electronic Engineers, Inc. (IEEE) in 1965 and by the United States of America Standards Institute in 1967 as a standard for all scientific and engineering literature.



1.2. Current and Voltage

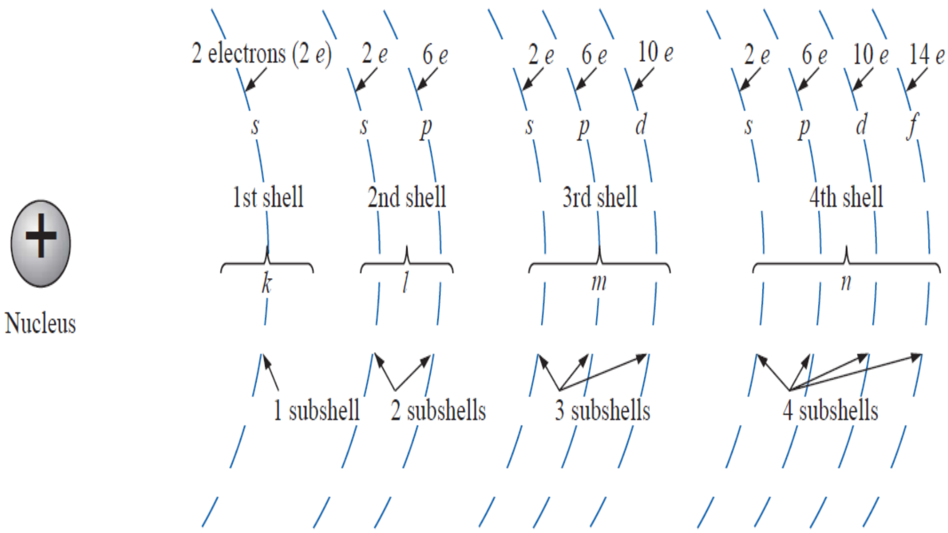
1.2.1 Introduction (JOHN DALTON)

A basic understanding of the fundamental concepts of current and voltage requires a degree of familiarity with the atom and its structure. The simplest of all atoms is the hydrogen atom, made up of two basic particles, the proton and the electron. The nucleus of the hydrogen atom is the proton, a positively charged particle. The orbiting electron carries a negative charge that is equal in magnitude to the positive charge of the proton. In all other elements, the nucleus also contains neutrons, which are slightly heavier than protons and have no electrical charge. The helium atom, for example, has two neutrons in addition to two electrons and two protons. *In all*

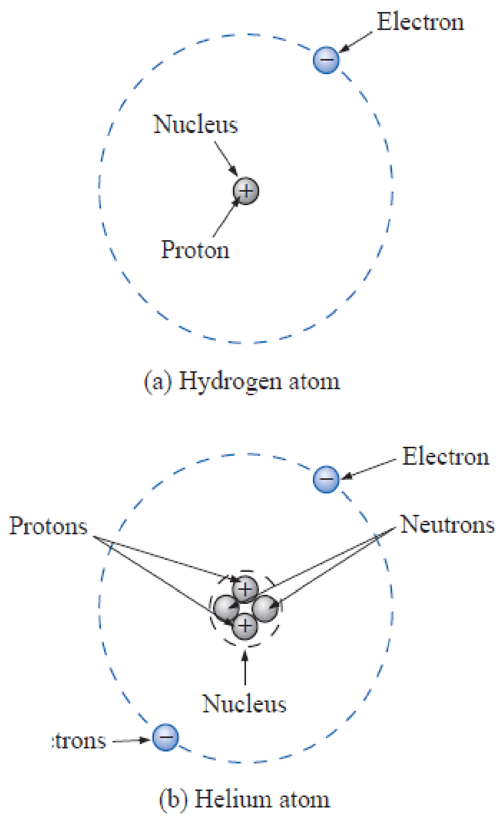
neutral atoms the number of electrons is equal to the number of protons. **The mass of the electron is 9.11×10^{-28} g, and that of the proton and neutron is 1.672×10^{-24} g.**

Different atoms will have various numbers of electrons in the concentric shells about the nucleus. The first shell, which is closest to the nucleus, can contain only two electrons. If an atom should have three electrons, the third must go to the next shell. The second shell can contain a maximum of eight electrons; the third, 18; and the fourth, 32; as determined by the equation $2n^2$, where n is the shell number. These shells are usually denoted by a number (n = 1, 2, 3, . . .) or letter (n = *k, l, m, . . .*).

Each shell is then broken down into subshells, where the first subshell can contain a maximum of two electrons; the second subshell, six electrons; the third, 10 electrons; and the fourth, 14. The subshells are usually denoted by the letters *s, p, d,* and *f,* in that order, outward from the nucleus.



Shells and subshells of the atomic structure.

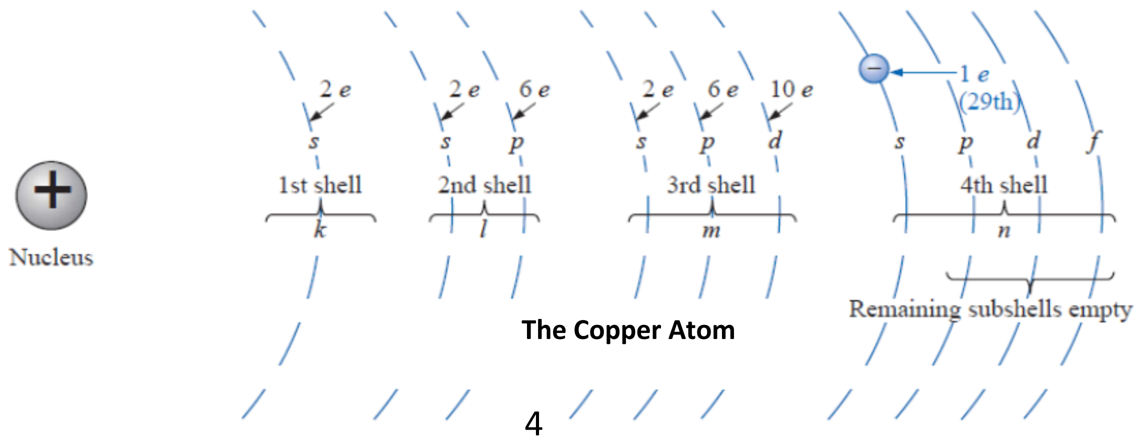


It has been determined by experimentation that unlike charges attract, and like charges repel. The force of attraction or repulsion between two charged bodies Q1 and Q2 can be determined by Coulomb's law:

$$F\ (\text{attraction or repulsion}) = \frac{kQ_1Q_2}{r^2} \qquad \text{(Newtons)}$$

Where F is in newton, k=$9\times10^{9}\,Nm^2$/C, $Q1$ and $Q2$ are the charges in coulombs, and r is the distance in meters between the two charges.

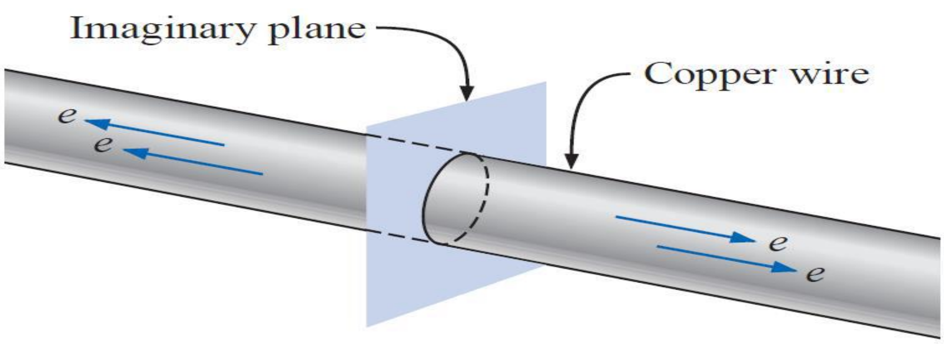
Copper is the most commonly used metal in the electrical/electronics industry. An examination of its atomic structure will help identify why it has such widespread applications. The copper atom has one more electron than needed to complete the first three shells. This incomplete outermost subshell, possessing only one electron, and the distance between this electron and the nucleus reveal that the twenty-ninth electron is loosely bound to the copper atom. If this twenty-ninth electron gains sufficient energy from the surrounding medium to leave its parent atom, it is called a free electron. In one cubic inch of copper at room temperature, there are approximately 1.4×10^{24} free electrons.



The Copper Atom

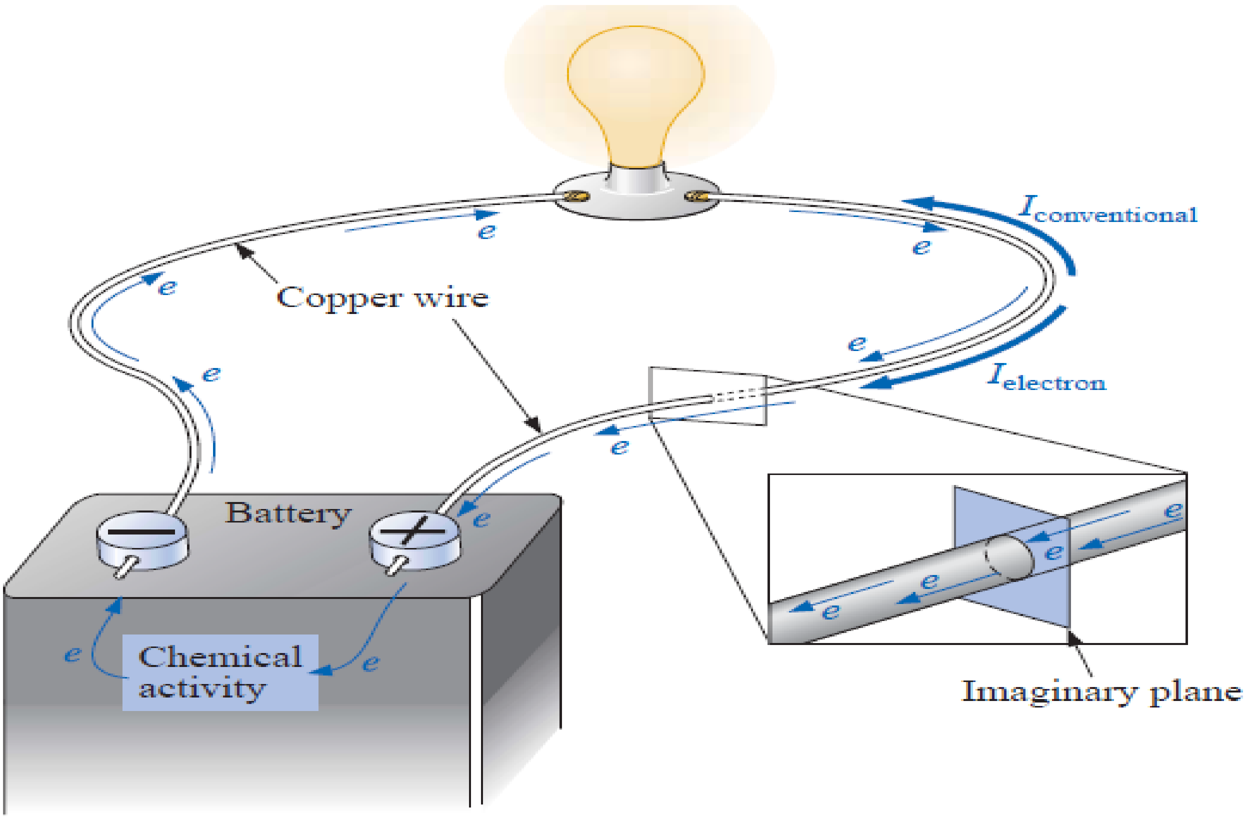
1.2.2. CURRENT

Consider a short length of copper wire cut with an imaginary perpendicular plane, producing the circular cross section. At room temperature with no external forces applied, there exists within the copper wire the random motion of free electrons created by the thermal energy that the electrons gain from the surrounding medium. When atoms lose their free electrons, they acquire a net positive charge and are referred to as positive ions. The free electrons are able to move within these positive ions and leave the general area of the parent atom, while the positive ions only oscillate in a mean fixed position. For this reason, *the free electron is the charge carrier in a copper wire or any other solid conductor of electricity*.



Random motion of electrons in a copper wire with no external "pressure" (voltage) applied.

Let us now connect copper wire between two battery terminals and a light bulb, to create the simplest of electric circuits. The battery, at the expense of chemical energy, places a net positive charge at one terminal and a net negative charge on the other. The instant the final connection is made, the free electrons (of negative charge) will drift toward the positive terminal, while the positive ions left



behind in the copper wire will simply oscillate in a mean fixed position. The negative terminal is a “supply” of electrons to be drawn from when the electrons of the copper wire drift toward the positive terminal.

The chemical activity of the battery will absorb the electrons at the positive terminal and will maintain a steady supply of electrons at the negative terminal. The flow of charge (electrons) through the bulb will heat up the filament of the bulb through friction to the point that it will glow red hot and emit the desired light. If 6.242×10^{18}*electrons* drift at uniform velocity through the imaginary circular cross section in 1 *second,* the flow of charge, or *current,* is said to be 1 ampere (A) in honor of André Marie Ampère.

In electric circuit, the charge is often carried by moving electrons in the wire. Therefore, electric current are follows of electric charge. The electric current is defined to be the rate at which charge flow across any cross sectional area. If an amount of charge ΔQ throughout a surface in a time interval Δt, then the average current I_{av} is given by:

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

The current in amperes can now be calculated using the following equation:

$$I = \frac{Q}{t}$$

I = amperes (A)
Q = coulombs (C)
t = seconds (s)

Example 1

The charge flowing through the imaginary surface is 0.16 C every 64 ms. Determine the current in amperes.

Example 2:

Determine the time required for 4×10^{16} electrons to pass through the imaginary surface if the current is 5 mA.(electron charge 1.602×10^{-19} *Colomb*) .

1.2.2.1. Current Density

It is about how much current is following across the given area and mathematically can be written as:

$$J = \frac{I}{A}$$

Example 3:

A copper wire of are 5mm² has a current of 5mA following through it. Calculate the current density?

1.2.3 Resistance

The flow of charge through any material encounters an opposing force similar in many respects to mechanical friction. This opposition, due to the collisions between electrons and between electrons and other atoms in the material, *which converts electrical energy into another form of energy such as heat,* is called the resistance of the material. The unit of measurement of resistance is the ohm, for which the symbol is Ω, the capital Greek letter omega.

The resistance of any material with a uniform cross-sectional area is determined by the following four factors:

1. ***Material***
2. ***Length***
3. ***Cross-sectional area***
4. ***Temperature***

At a fixed temperature of 20°C (room temperature), the resistance is related to the other three factors by

$$R = \rho \frac{l}{A}$$

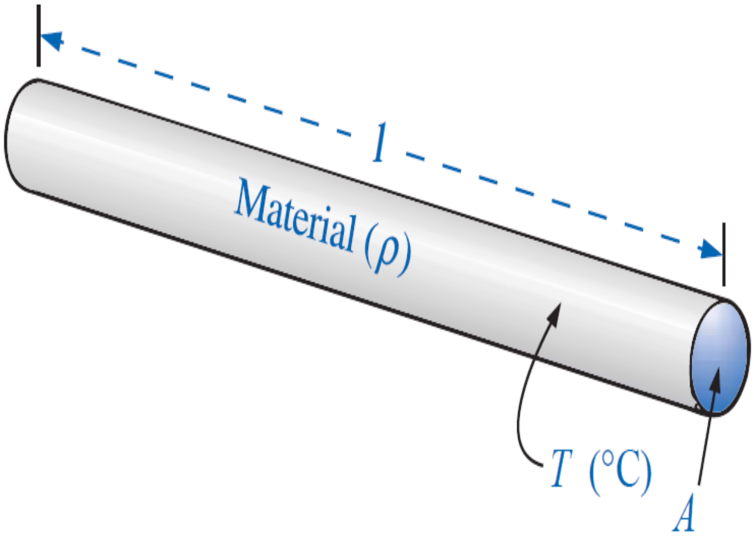
Where ρ (Greek letter rho) is a characteristic of the material called the resistivity, *l* is the length of the sample, and *A* is the cross-sectional area of the sample.

1.2.3.1 RESISTANCE: CIRCULAR WIRE

The resistivity ρ is also measured in ohms per mil-foot, or *ohm-meters* in the SI system of units. Some typical values of ρ are:

Resistivity (ρ) of various materials.

Material	ρ @ 20°C
Silver	9.9
Copper	10.37
Gold	14.7
Aluminum	17.0
Tungsten	33.0
Nickel	47.0
Iron	74.0
Constantan	295.0
Nichrome	600.0
Calorite	720.0
Carbon	21,000.0



For circular wires, the quantities $\rho, l, and\ A$ have the following units:

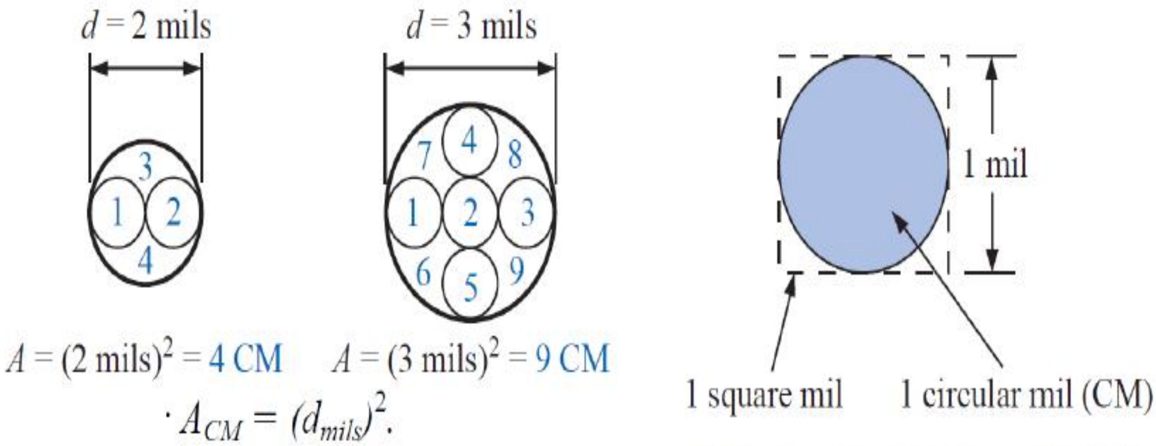
ρ: CM-ohms/ft at $T = 20°C$
l: feet
A: circular mils (CM)

Note that the area of the conductor is measured in circular mils (CM) and *not* in *square meters, inches,* and so on, as determined by the equation:

$$\text{Area (circle)} = \pi r^2 = \frac{\pi d^2}{4}$$

r = radius
d = diameter

A wire with a diameter of 1 mil has an area of 1 circular mil (CM), the area in circular mils is simply equal to the diameter in mils square; that is: $A_{CM} = (d_{mils})^2$



Defining the circular mil (CM).

EXAMPLE 4

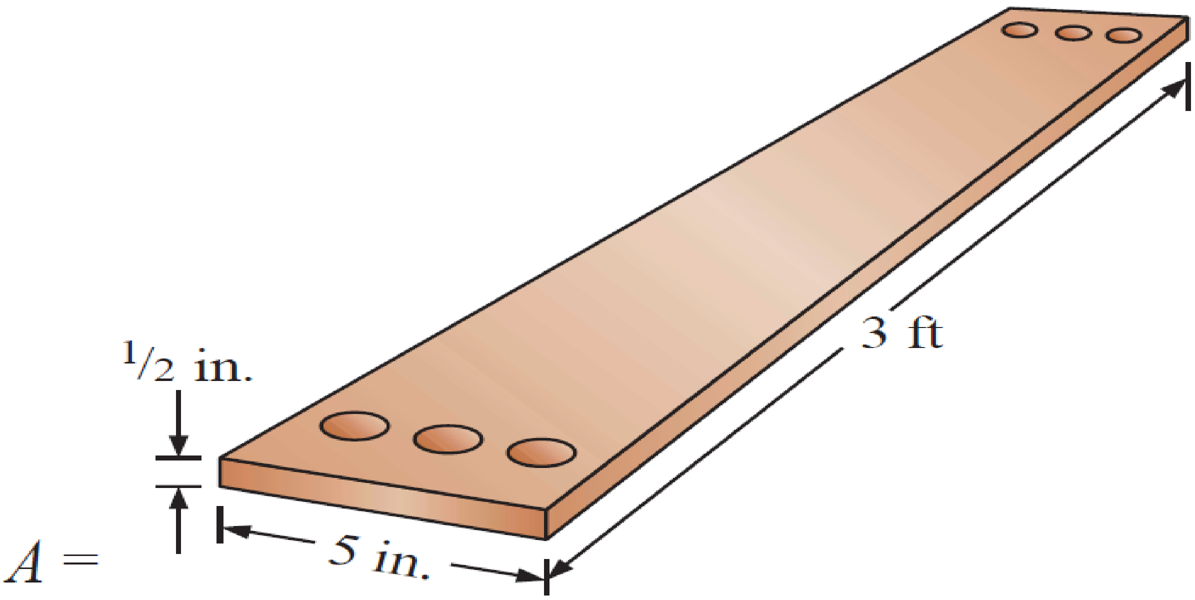
What is the resistance of a 100-ft length of copper wire with a diameter of 0.020 in. at 20°C?

EXAMPLE 5

An undetermined number of feet of wire have been used. Find the length of the remaining copper wire if it has a diameter of 1/16 in. and a resistance of 0.5 Ω.

EXAMPLE 6

What is the resistance of a copper bus-bar, as used in the power distribution panel of a high-rise office building, with the dimensions indicated in Fig. below?



1.2.3.2 WIRE TABLES

The wire table was designed primarily to standardize the size of wire produced by manufacturers throughout the United

States. As a result, the manufacturer has a larger market and the consumer knows that standard wire sizes will always be available. The table was designed to assist the user in every way possible; it usually includes data such as the cross-sectional area in circular mils, diameter in mils, ohms per 1000 feet at 20°C, and weight per 1000 feet. The American Wire Gage (AWG) sizes are given in Table below for solid round copper wire. A column indicating the maximum allowable current in amperes, as determined by the National Fire Protection Association, has also been included.

American Wire Gage (AWG) sizes.

	AWG #	Area (CM)	Ω/1000 ft at 20°C	Maximum Allowable Current for RHW Insulation (A)*
(4/0)	0000	211,600	0.0490	230
(3/0)	000	167,810	0.0618	200
(2/0)	00	133,080	0.0780	175
(1/0)	0	105,530	0.0983	150
	1	83,694	0.1240	130
	2	66,373	0.1563	115
	3	52,634	0.1970	100
	4	41,742	0.2485	85
	5	33,102	0.3133	—
	6	26,250	0.3951	65
	7	20,816	0.4982	—
	8	16,509	0.6282	50
	9	13,094	0.7921	—
	10	10,381	0.9989	30
	11	8,234.0	1.260	—
	12	6,529.0	1.588	20
	13	5,178.4	2.003	—
	14	4,106.8	2.525	15
	15	3,256.7	3.184	
	16	2,582.9	4.016	
	17	2,048.2	5.064	
	18	1,624.3	6.385	
	19	1,288.1	8.051	
	20	1,021.5	10.15	
	21	810.10	12.80	
	22	642.40	16.14	
	23	509.45	20.36	
	24	404.01	25.67	
	25	320.40	32.37	
	26	254.10	40.81	
	27	201.50	51.47	
	28	159.79	64.90	
	29	126.72	81.83	
	30	100.50	103.2	
	31	79.70	130.1	
	32	63.21	164.1	
	33	50.13	206.9	
	34	39.75	260.9	
	35	31.52	329.0	
	36	25.00	414.8	
	37	19.83	523.1	
	38	15.72	659.6	
	39	12.47	831.8	
	40	9.89	1049.0	

EXAMPLE 8

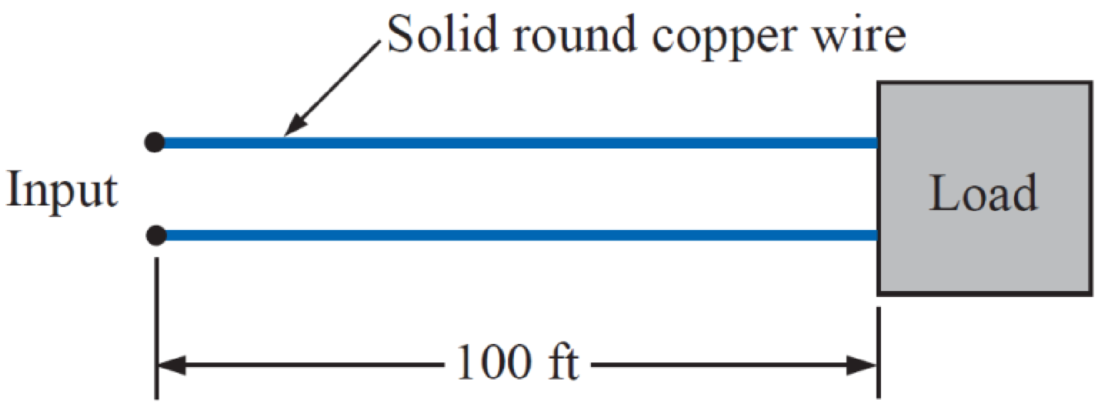
Find the resistance of 650 ft of #8 copper wire (*T* =20°C).

EXAMPLE 9

What is the diameter, in inches, of a #12 copper wire?

EXAMPLE 10

For the system of Fig. below, the total resistance of *each* power line cannot exceed 0.025 Ω, and the maximum current to be drawn by the load is 95 A. What gage wire should be used?



1.2.3.3 RESISTANCE: METRIC UNITS

The design of resistive elements for various areas of application, including thin-film resistors and integrated circuits, uses metric units for the quantities. In SI units, the resistivity would be measured in ohm-meters, the area in square meters, and the length in meters. However, the meter is generally too large a unit of measure for most applications, and so the centimeter is usually employed. The resulting dimensions are therefore

ρ:	ohm-centimeters
l:	centimeters
A:	square centimeters

Table below provides a list of values of r in ohm-centimeters.

Resistivity (ρ) of various materials in ohm-centimeters.

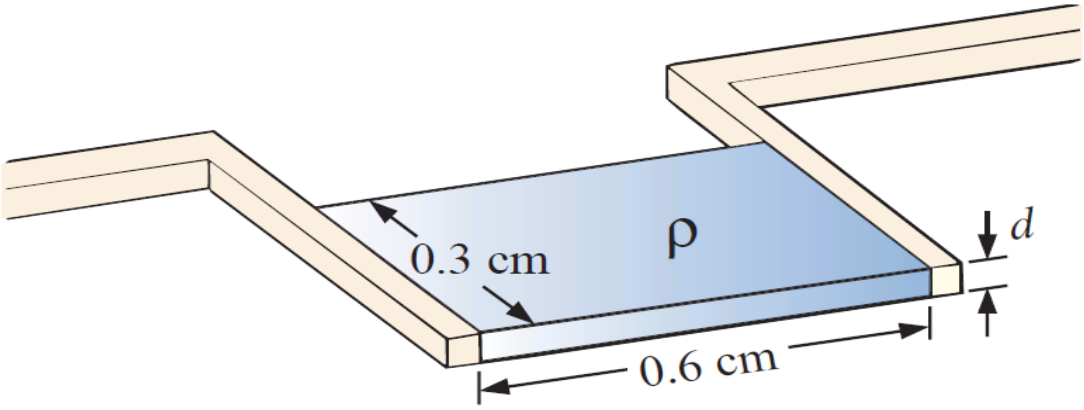
Silver	1.645×10^{-6}
Copper	$\mathbf{1.723 \times 10^{-6}}$
Gold	2.443×10^{-6}
Aluminum	2.825×10^{-6}
Tungsten	5.485×10^{-6}
Nickel	7.811×10^{-6}
Iron	12.299×10^{-6}
Tantalum	15.54×10^{-6}
Nichrome	99.72×10^{-6}
Tin oxide	250×10^{-6}
Carbon	3500×10^{-6}

EXAMPLE 11

Determine the resistance of 100 ft of #28 copper telephone wire if the diameter is 0.0126 in.

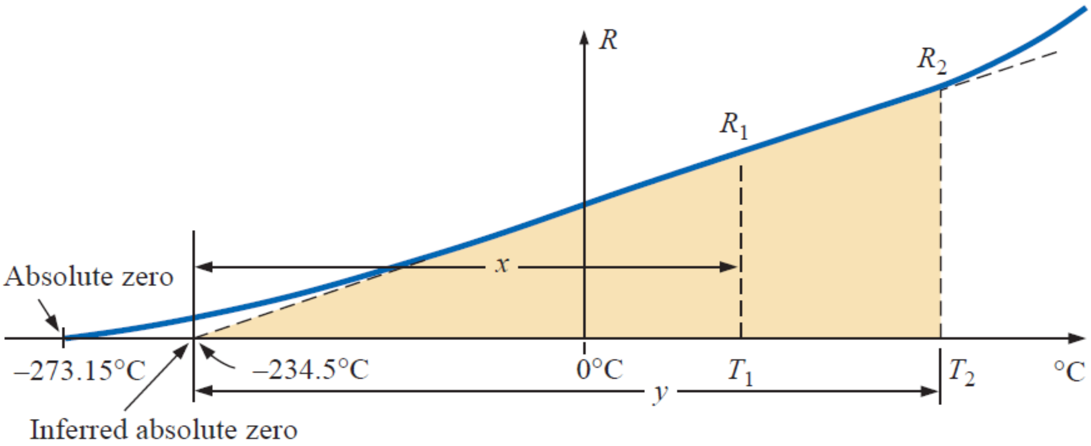
EXAMPLE 12

Determine the resistance of the thin-film resistor of Fig. below if the sheet resistance *Rs* (defined by *Rs* =r/*d*) is 100 Ω.



1.2.3.4. TEMPERATURE EFFECTS

Temperature has a significant effect on the resistance of conductors, semiconductors, and insulators.



Effect of temperature on the resistance of copper.

$$\frac{x}{R1} = \frac{y}{R2}$$

OR

$$\frac{234.5 + T1}{R1} = \frac{234.5 + T2}{R2}$$

EXAMPLE 13

If the resistance of a copper wire is 50 Ω at 20°C, what is its resistance at 100°C (boiling point of water)?

EXAMPLE 14

If the resistance of a copper wire at freezing (0°C) is 30 Ω, what is its resistance at -40°C?

EXAMPLE 15

If the resistance of aluminum wire at room temperature (20°C) is 100 mΩ (measured by a mille-ohmmeter), at what temperature will its resistance increase to 120 mΩ?

1.2.3.5 Temperature Coefficient of Resistance

There is a second popular equation for calculating the resistance of a conductor at different temperatures.

$$\boldsymbol{\alpha_{20} = \frac{1}{|T1| + 20}}$$

as the temperature coefficient of resistance at a temperature of 20°C, and *R*20 as the resistance of the sample at 20°C, the resistance *R*1 at a temperature *T*1 is determined by:

$$\boldsymbol{R1 = R1[1 + \alpha_{20}(T1 - 20)}$$

Temperature coefficient of resistance for various conductors at 20°C.

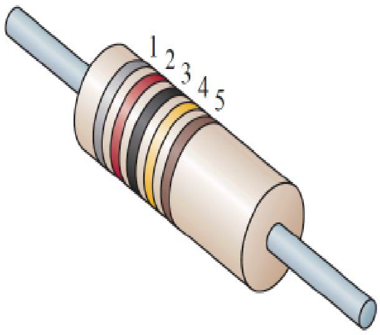
Material	Temperature Coefficient (α_{20})
Silver	0.0038
Copper	**0.00393**
Gold	0.0034
Aluminum	0.00391
Tungsten	0.005
Nickel	0.006
Iron	0.0055
Constantan	0.000008
Nichrome	0.00044

1.2.3.6. COLOR CODING AND STANDARD RESISTOR VALUE

A wide variety of resistors, fixed or variable, are large enough to have their resistance in ohms printed on the casing. Some, however, are too small to have numbers printed on them, so a system of color coding is used. For the fixed molded composition resistor, four or five color bands are printed on one end of the outer casing

Resistor color coding.

Bands 1–3*	Band 3	Band 4	Band 5
0 Black	0.1 Gold } multiplying factors	5% Gold	1% Brown
1 Brown	0.01 Silver } multiplying factors	10% Silver	0.1% Red
2 Red		20% No band	0.01% Orange
3 Orange			0.001% Yellow
4 Yellow			
5 Green			
6 Blue			
7 Violet			
8 Gray			
9 White			



EXAMPLE 16

Find the range in which a resistor having the following color bands must exist to satisfy the manufacturer's tolerance:

a. 1st band	2nd band	3rd band	4th band	5th band
Gray	Red	Black	Gold	Brown
8	2	0	5%	1%

b. 1st band	2nd band	3rd band	4th band	5th band
Orange	White	Gold	Silver	No color
3	9	0.1	10%	

1.2.4 VOLTAGE

The flow of charge is established by an external "pressure" derived from the energy that a mass has by virtue of its position: potential energy. Energy, by definition, is the capacity to do work. If a mass (m) is raised to some height (h) above a reference plane, it has a measure of potential energy expressed in joules (J) that is determined by

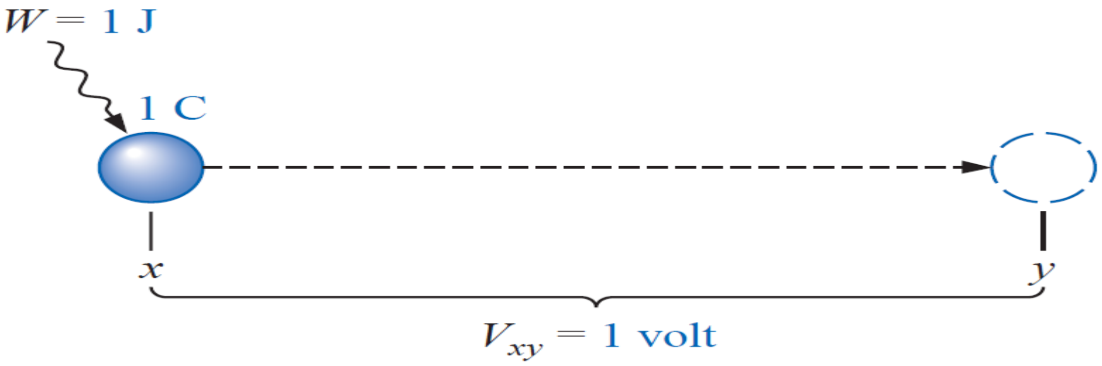
$$W = mgh \ Joules.(J)$$

Where *g* is the gravitational acceleration (9.754 m ⁄s^2). This mass now has the "potential" to do work such as crush an object placed on the reference plane.

In the battery, the internal chemical action will establish (through an expenditure of energy) an accumulation of negative charges (electrons) on one terminal (the negative terminal) and positive charges (positive ions) on the other (the positive terminal). A "positioning" of the charges has been established that will result in a potential difference between the terminals. If a conductor is connected between the terminals of the battery, the electrons at the negative terminal have sufficient potential energy to overcome collisions with other particles in the conductor and the repulsion from similar charges to reach the positive terminal to which they are attracted.

A potential difference of 1 volt (V) exists between two points if 1 joule (J) of energy is exchanged in moving 1 coulomb (C) of charge between the two points.

The unit of measurement volt was chosen to honor Alessandro Volta. Pictorially, if one joule of energy (1 J) is required to move the one coulomb (1 C) of charge of Fig. 2.10 from position *x* to position *y*, the potential difference or voltage between the two points is one volt (1 V).



Defining the unit of measurement for voltage.

a potential difference or voltage is always measured between two points in the system. Changing either point may change the potential difference between the two points under investigation.

In general, the potential difference between two points is determined by

$$V = \frac{W}{Q}$$

Example 17

Find the potential difference between two points in an electrical system if 60 J of energy are expended by a charge of 20 C between these two points.

Example 18

Determine the energy expended moving a charge of 50 mC through a potential difference of 6 V. Determine the energy expended moving a charge of 50 mC through a potential difference of 6 V.

Example 19

Find the voltage drop from the point a to point b, if 24J are required to move charge of 3C from point a to point b.

1.2.5 Power

Is an indication of how much work can be accomplished in a specific amount of time, that is a rate of doing work.

Power measure in watt (W), and work in Joule (J).

$$\boldsymbol{P = \frac{W}{t}}$$

1 hours power hp=746 watt

$$\boldsymbol{P = \frac{W}{t} = \frac{QV}{t} = IV}$$

Example 20

Find the power delivered to the d.c motor if the voltage applied is 120 v and the current equal to 5A.

1.2.6. Energy

Electric energy used or produced is the product of the electric power and the time

W (kilo watt hours) =P (kilo watt) x t (hour) (Joules)

Example 21

For the dial positions reading 5360, calculate the electricity bill if the previous reading was 4650 Kwh and the average coast is 7 € per kilo watt hour.

Example 22

What is the total coast of using the following loads at 7 € per kilo watt hour.

a- 1200 w toaster for 30 min

b- Six 50 w bulb for 4 h.

c- 400 w washing machine for 45 min

d- 4800 w electric clothes dryer for 20 min

1.2.7 Efficiency

Any electrical systems that convert energy from one form to another can be represented as

Input energy=output energy+ energy stored in the system or lost

$$\frac{W_{in}}{t} = \frac{W_{out}}{t} + \frac{W_{lost}}{t}$$

$$P_{in} = P_{out} + P_{lost}$$

$$Efficiency = \eta = \frac{output\ power}{input\ power} \times 100\%$$

Example 23

A 2 hp motor operates at an efficiency of 75%, what is the power input in watt? If the input current is 9.05 A what is the input voltage?

Fundamentals of Electrical Engineering.

UNIVERSITY OF ANBAR/MECHANICAL ENG. DEP.

FIRST LEVEL/Second Semester/2019-2020

Dr. Falah Shallal Khalifa